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ANALYTICAL TARGET CASCADING IN AUTOMOTIVE VEHICLE DESIGN

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ABSTRACT

Target cascading in product development is a systematic effort to propagate the desired top-level system design targets to appropriate specifications for subsystems and components in a consistent and efficient manner. If analysis models are available to represent the relevant design decisions, analytical target cascading can be formalized as a hierarchical multilevel optimization problem. The article demonstrates this complex modeling and solution process in the chassis design of a sport-utility vehicle. Ride quality and handling targets are cascaded down to systems and subsystems utilizing suspension, tire, and spring analysis models. Potential incompatibilities among targets and constraints throughout the entire system can be uncovered and the trade-offs involved in achieving system targets under different design scenarios can be quantified.

NOMENCLATURE

$C_{\alpha f}$	tire lateral cornering stiffnesses for front
$C_{\alpha r}$	tire lateral cornering stiffnesses for rear
K_{sf}	stiffness of front suspensions
K_{sr}	stiffness of rear suspensions
K_{tf}	stiffness of front tires
K_{tr}	stiffness of rear tires
P_{if}	front tire inflation pressure
P_{ir}	rear tire inflation pressure
P_o	original design optimization problem
P_v	vehicle level target cascading optimization problem
P_s	system level target cascading optimization problem
P_{ss}	subsystem level target cascading optimization problem
$\mathbf{R}^L$	target values of **R** from a lower level
$\mathbf{R}^U$	target values of **R** from an upper level
$\mathbf{R}$	responses computed by analysis models
$\mathbf{T}$	design targets
a	distance from vehicle center of mass to front axle
b	distance from vehicle center of mass to rear axle
f	objective for the design problem
$\mathbf{g}$	inequality constraints for the design problem
$\mathbf{h}$	equality constraints for the design problem
k_{us}	understeer gradient
r	response function
u	vehicle forward velocity
$\mathbf{x}$	vector of all design variables $(\tilde{\mathbf{x}}, \mathbf{y})$
$\tilde{\mathbf{x}}$	local design variables
$\mathbf{x}^{min}$	lower bound of **x**
$\mathbf{x}^{max}$	upper bound of **x**
$\mathbf{y}$	linking design variables
$\mathbf{y}^L$	target values of **y** from a lower level
$\mathbf{y}^U$	target values of **y** from an upper level
$\varepsilon_{\mathbf{R}}$	target deviation tolerance for responses
$\varepsilon_{\mathbf{y}}$	target deviation tolerance for linking variables
ω_p	pitch natural frequency
ω_{sf}	first natural frequency of front suspension
ω_{sr}	first natural frequency of rear suspension
ω_{tf}	second natural frequency (wheel hop frequency) of front suspension
ω_{tr}	second natural frequency (wheel hop frequency) of rear suspension
$zsmax$	suspension deflection at jounce bumper contact

1. INTRODUCTION

The product development process for complex artifacts is most effective when the required design tasks can be accomplished in a concurrent and consistent manner. Concurrency means that individual design tasks are conducted separately, and consistency means that key links identified among different design tasks are observed and enforced until the concurrent design process yields a final product. The target cascading process attempts to achieve this consistency and concurrency early in the development process [Kim et al. 2000, Kim 2001]. The important specifications or “targets” for the entire system (as well as for each subsystem and component) are identified first, specifically those that will influence other parts of the system. These targets are then propagated or “cascaded” to the rest of the system and appropriate values are assigned for the expected performance of each element of the system. The actual design tasks are then executed locally for each individual element, and interaction with the rest of the system is revisited only when a target cannot be met. When the design decisions can be modelled analytically, the process can be formalized as a multilevel optimization problem referred to as analytical target cascading. The formulation and solution of this problem is a complex task. Much of the motivation for the work described in this article comes from a need to demonstrate how target cascading will work for a problem of realistic complexity, such as an automotive vehicle.

Multilevel optimization methods have been well studied [e.g., Sobieski et al. 1987, Cramer et al. 1994]. Collaborative optimization [Braun 1996, Braun et al. 1996, Tappeta and Renaud 1997] is particularly interesting in the present context. In this formulation design objectives in the subproblems attempt to minimize the discrepancy between the interaction variables and the targets, and should become zero at the optimum. Constraints in the original optimization problem are distributed in the subsystem optimization problems, and subproblem objectives become equality constraints at the system level. During iterations, subproblems may return different values for an interdisciplinary variable, which can cause convergence difficulties in that equality constraints at the system level are not satisfied [Alexandrov 2000]. Convergence difficulties are not uncommon for the coordination strategies needed to solve multilevel optimization problems. Though different from collaborative optimization, target cascading shares the idea of minimizing deviations between design problems to achieve consistency but can be shown to satisfy constraint qualifications [Kim 2001]. In collaborative optimization, analysis models are decomposed at the same level and a coordination problem is defined on top of the bilevel modeling hierarchy. Without a convergent coordination strategy, it is not clear to extend collaborative optimization in multilevel hierarchy. In target cascading, multilevel optimization problem is formulated to enable multidisciplinary decision making in multiple levels. Non-ascent property of the hierarchical overlapping coordination is utilized to demonstrate non-ascent property of target cascading coordination [Michelena et al. 1999, Park et al. 2000, Kim 2001]. In the present study, models are checked for feasibility and boundedness [Papalambros and Wilde 2000] and for constraint qualifications of the additional deviation constraints [Bazaraa et al. 1993].

The next section reviews briefly the basic concepts in the formal target cascading process. A chassis design problem is then outlined, its constituent models are developed, and the mathematical problem is posed. Solution of this problem shows how top-level targets can be cascaded to derive subsystem and component specifications. Such a capability is shown to be an effective early product development tool: Trade-offs among desired top-level target values can be quantitatively assessed, while incompatibilities can be uncovered and traced to design specifications or bounds at the subsystem and component levels.

2. SOME BASIC CONCEPTS IN TARGET CASCADING

The reader is referred to Kim et al. [2000] and Kim [2001] for a complete explanation of generic target cascading (TC) formulations. Here we draw attention to the distinction between the design and analysis models with which the hierarchy is constructed, and give the mathematical form of the TC problem.

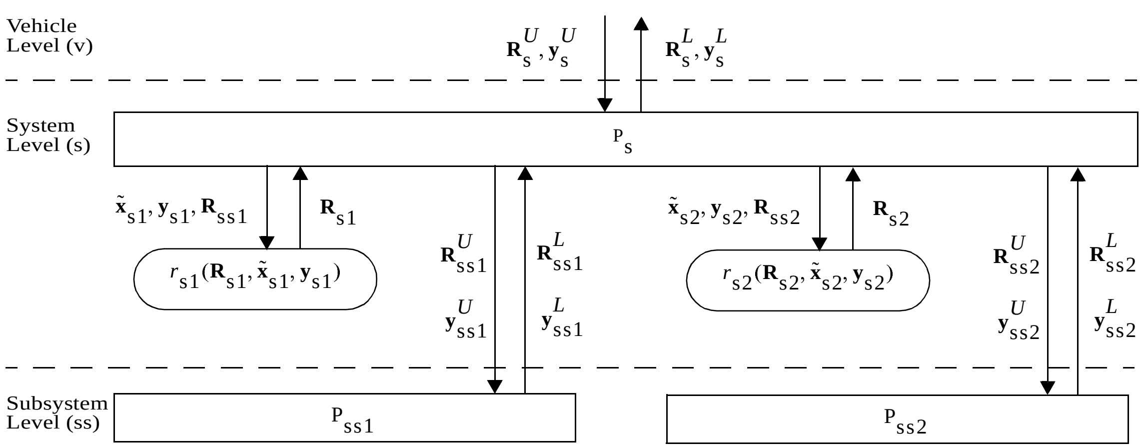

Figure 1. Flows from/into the system-level design problem

Modeling Hierarchy

The reader may refer to the IEEE Standards for multi-level systems engineering concepts for further description of partitioned design elements [IEEE 1998]. A complex problem, such as vehicle design, can be partitioned into a multilevel hierarchical structure. Two types of models exist in the modeling hierarchy of the TC process: *optimal design models* P and *analysis models r* [Kim et al. 2000]. Optimal design models call analysis models to evaluate vehicle, system, subsystem and component responses. Thus, analysis models take design variables and parameters, as well as lower level responses, and return responses for design problems. A response is defined as an output from an analysis model, and a linking variable is defined as a design variable common between two or more design problems.

Figure 1 shows interactions between analysis models and design models at the system level. Targets for system responses and system linking variables $\mathbf{R}_s^U$ and $\mathbf{y}_s^U$ are passed down from the vehicle level. After solving the system design problem, target values for system responses and system linking variables $\mathbf{R}_s^L$ and $\mathbf{y}_s^L$ are passed up to the vehicle level. Likewise, for subsystem 1, $\mathbf{R}_{ss1}^U$ and $\mathbf{y}_{ss1}^U$ are passed down as targets from the system-level design problem, whereas $\mathbf{R}_{ss1}^L$ and $\mathbf{y}_{ss1}^L$ are returned to the system level. Responses from subsystem 1, $\mathbf{R}_{ss1}$, system local design variables $\tilde{\mathbf{x}}_{s1}$, and system linking variables $\mathbf{y}_{s1}$ are input to the analysis model r_{s1}, whereas system responses $\mathbf{R}_{s1}$ are returned as output.

Mathematical Problem Statement of the Design Problem

The original design problem, in a vehicle context, can be stated as follows: find a design that minimizes the deviations between the overall design targets and responses, while satisfying all constraints. Alternatively, determine the values of vehicle, system, subsystem and component parameters that minimize the deviation of vehicle responses from vehicle targets. The original design problem P_0 is formally stated in Eq. (1).

The objective is defined as the discrepancy between the target **T** and the response **R** obtained from the analysis model $r(\mathbf{x})$; **g** and **h** are inequality and equality design constraint vectors with sizes m_i, m_e, and the design variable **x** is defined within lower and upper bounds, $\mathbf{x}^{min}$ and $\mathbf{x}^{max}$.

$$
\begin{aligned}
P_0:\ & \underset{\mathbf{x}}{\text{Minimize}} \quad \|\mathbf{T}-\mathbf{R}\| \\
& \text{where} \quad \mathbf{R} = r(\mathbf{x}) \\
& \text{subject to} \\
& g_i(\mathbf{x}) \le 0 \qquad i = 1, \ldots, m_i \\
& h_j(\mathbf{x}) = 0 \qquad j = 1, \ldots, m_e \\
& x_k^{min} \le x_k \le x_k^{max} \qquad k = 1, \ldots, n
\end{aligned}
\tag{1}
$$

3. A TARGET CASCADING PROCESS FOR VEHICLE RIDE AND HANDLING

In this section we give an overview of a TC model for the chassis system of a typical sport-utility vehicle (SUV) aimed at establishing vehicle ride and handling targets. The model is obviously simplified but retains sufficient complexity to be realistic. Figure 2 gives a schematic of the information flow in the vehicle design problem structure. Each block indicates an optimal design model where design decisions are made to achieve minimum deviation from the targets. Each design model calls one or more analysis models to evaluate the current design. The

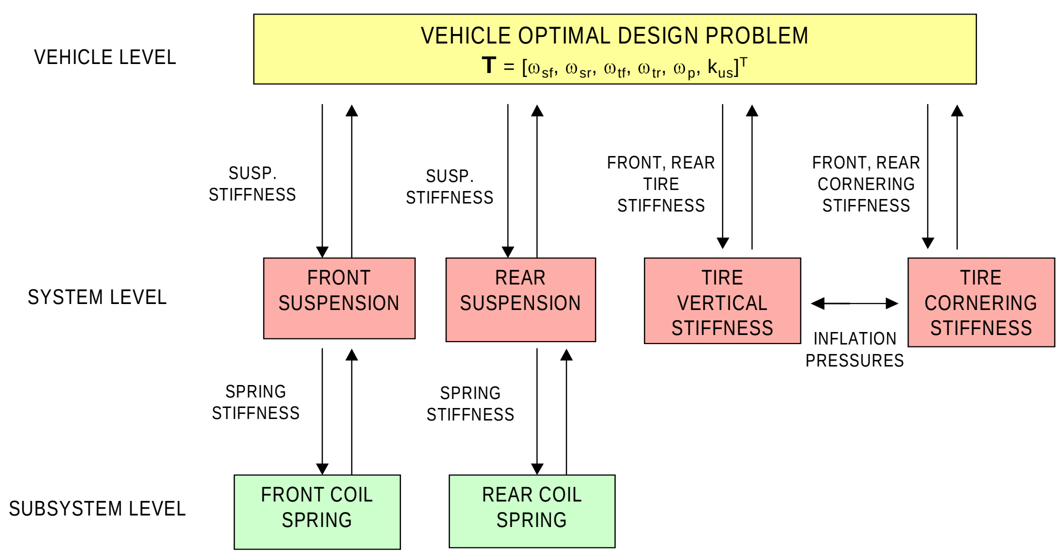

Figure 2. SUV chassis design problem structure

vehicle-level design problem contains two analysis models, a "half-car" model and a "bicycle" model. System-level analysis models for the front and rear suspensions are multibody-dynamics models of short-long arm (SLA) suspensions [Hogland 2000]. The tire models call the tire stiffness equations described in [Wong 1993].

The following vehicle-level targets are prescribed:

- first natural frequency of front and rear suspension (ω_{sf}, ω_{sr})
- second natural frequency (wheel hop frequency) of front and rear suspension (ω_{tf}, ω_{tr})
- pitch natural frequency (ω_p)
- understeer gradient (k_{us})

These six quantities constitute the target vector, for which the half-car and bicycle analysis models generate responses. The computed variable values are then cascaded to the system-level design problem as targets. For example, the front suspension stiffness is changed to achieve the desired first natural frequency for the front suspension. Once an optimal value of the stiffness is found at the vehicle design problem, that value becomes a *target* value at the system-level design problem, in which the suspension design variables (coil spring stiffness and free length) are altered to achieve a suspension configuration with a stiffness as close to the cascaded target value as possible. The computed values of the variables, such as the coil spring stiffness that gives the optimal suspension stiffness, are then cascaded to the subsystem level as targets. The spring subsystem variables are optimized to achieve minimal deviation from the targets assigned for the coil spring stiffness.

Similarly, optimal tire stiffness and cornering stiffness calculated at the vehicle level become targets at the system level, where system-level variables (tire inflation pressure) are changed to meet the stiffness targets. In tire design models for vertical and cornering stiffnesses, the inflation pressure is common, i.e., the inflation pressure is a linking variable.

Once the vehicle design targets are cascaded down to the lowest level, the resulting design information must then be passed back to higher levels, up to the top level. In general, it will not be possible to achieve the target values exactly in each design problem, due to constraints and variable bounds or due to lower level responses. For example, the front suspension stiffness obtained from the system-level optimization problem might not match the target value from the vehicle level due to constraints on coil spring free length and stiffness. Similarly, upon cascading the desired coil spring stiffness to the coil spring component design problem, packaging or fatigue constraints might result in spring stiffnesses deviating from the specified target value. Deviation in spring coil stiffness will subsequently result in a deviation of the overall suspension stiffness, which in turn will affect the first ride frequency of the vehicle. Thus an iterative process working in both a top-down and a bottom-up fashion will lead to a consistent design and/or uncover potential incompatibilities among overall system responses, targets, and element parameters.

4. MATHEMATICAL PROBLEM STATEMENT AND MODEL DEVELOPMENT

The full TC model is presented in this section. At each level, we present the general form of the TC model and then its instantiation to the problem at hand.

The TC process does not require high fidelity models. Rather, it requires models that capture only the influence of design variables and responses in each system element that would affect other parts of the system. Indeed finding models of *appropriate* fidelity is a practical challenge in the execution of the TC process.

Vehicle Level

At the top level of the vehicle hierarchy the problem is stated as follows:

$$
\begin{aligned}
&P_v: \quad \underset{\tilde{\mathbf{x}}_v,\, \mathbf{y}_s,\, \mathbf{R}_s,\, \varepsilon_{\mathbf{R}},\, \varepsilon_{\mathbf{y}}}{\text{Minimize}} \ \left\|\mathbf{R}_v - \mathbf{T}_v\right\| + \varepsilon_{\mathbf{R}} + \varepsilon_{\mathbf{y}} \\
&\text{where} \quad \mathbf{R}_v = r_v(\mathbf{R}_s, \tilde{\mathbf{x}}_v) \\
&\text{subject to} \\
&\quad \left\|\mathbf{R}_s - \mathbf{R}_s^L\right\| \le \varepsilon_{\mathbf{R}}, \quad \left\|\mathbf{y}_s - \mathbf{y}_s^L\right\| \le \varepsilon_{\mathbf{y}} \\
&\quad \mathbf{g}_v(\mathbf{R}_v, \tilde{\mathbf{x}}_v) \le \mathbf{0},\ \mathbf{h}_v(\mathbf{R}_v, \tilde{\mathbf{x}}_v) = \mathbf{0} \\
&\quad \tilde{\mathbf{x}}_v^{min} \le \tilde{\mathbf{x}}_v \le \tilde{\mathbf{x}}_v^{max}
\end{aligned}
\tag{2}
$$

The objective that minimizes deviations between design targets $\mathbf{T}_v$ and vehicle responses $\mathbf{R}_v$ is modified by adding deviation tolerances $\varepsilon_{\mathbf{R}}$ and $\varepsilon_{\mathbf{y}}$ to coordinate values of the responses from the system, $\mathbf{R}_s$, and the system linking variables, $\mathbf{y}_s$. If there are p subproblems, then $\mathbf{R}_s = \mathbf{R}_{s1} \cup \ldots \cup \mathbf{R}_{sp}$ and $\mathbf{R}_{si} \cap \mathbf{R}_{sj} = \varnothing$ for $i \neq j$. System linking variables are defined as common design variables at the system level. Hence, we define

$$\left\|\mathbf{y}_s - \mathbf{y}_s^L\right\| \equiv \Psi(\left\|\mathbf{y}_s - \mathbf{y}_{s1}^L\right\|, \ldots, \left\|\mathbf{y}_s - \mathbf{y}_{si}^L\right\|, \ldots, \left\|\mathbf{y}_s - \mathbf{y}_{sp}^L\right\|) \tag{3}$$

where Ψ is essentially a function for averaging and $\mathbf{y}_{si}^L$ is a system linking variable calculated at the system optimal design problem *i*. One instance of Ψ can be

$$\left\|\mathbf{y}_s - \mathbf{y}_s^L\right\| \equiv \frac{1}{p}(\left\|\mathbf{y}_s - \mathbf{y}_{s1}^L\right\| + \ldots + \left\|\mathbf{y}_s - \mathbf{y}_{si}^L\right\| + \ldots + \left\|\mathbf{y}_s - \mathbf{y}_{sp}^L\right\|). \tag{4}$$

At convergence, the deviation tolerance becomes zero as the system linking variables converge to the same values for the different systems. The values of the system responses match $\mathbf{R}_S^L$, where $\mathbf{R}_S^L$ is the target response calculated at the system optimal design problem. Finally, $\mathbf{g}_v$ and $\mathbf{h}_v$ are inequality and equality design constraints at the vehicle level, subsets of the original constraints $\mathbf{g}$ and $\mathbf{h}$.

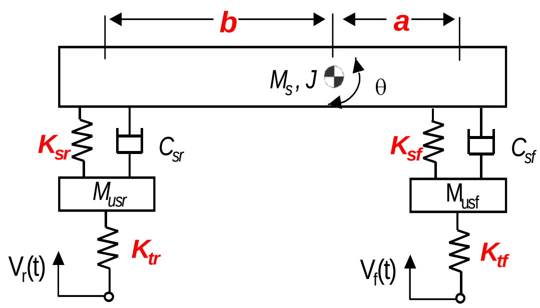

Figure 3. Half-Car Model

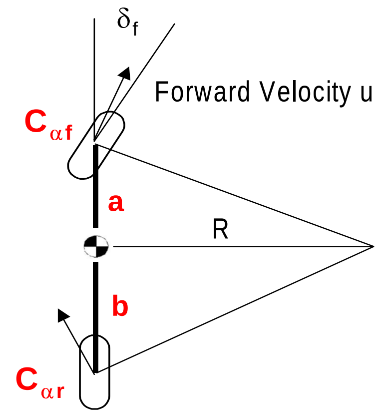

Figure 4. Cornering of a bicycle model

The five ride quality targets involve the half-car model of Figure 3. The target frequencies can be calculated in closed form as functions of sprung mass (M_s), front and rear unsprung masses (M_{usf}, M_{usr}), and suspension stiffnesses. The sprung and unsprung masses are assumed to be prescribed *a priori*, and are fixed design parameters. The vehicle body is treated as a single rigid body mass. Table 1 gives a summary of the vehicle-level variables, responses, and system-level linking variables and responses corresponding to the TC formulation at the vehicle level in Eq. (2).

The first natural frequencies of the suspensions are primarily affected by changing the front and rear suspension stiffnesses, and to a lesser extent by modifying the distances *a* and *b* from the center of gravity to the axles.

The handling target is the understeer gradient k_{us}, a measure of the magnitude and direction of the steering input for a vehicle to track a curve of constant radius *R* with forward velocity *u*. For the purpose of understeer analysis, it is convenient to represent the vehicle by the bicycle model shown in Figure 4. The understeer gradient is a function of *a* and *b* and of the front and rear tire lateral cornering stiffnesses $C_{\alpha f}$ and $C_{\alpha r}$.

Table 1: Summary of responses and variables at the vehicle level

Design problem	P_v
Responses ($\mathbf{R}_v$)	$\omega_{sf}, \omega_{sr}, \omega_{tf}, \omega_{tr}, \omega_p, k_{us}$
Local variables ($\tilde{\mathbf{x}}_v$)	a, b
System-level linking variables ($\mathbf{y}_s$)	P_{if}, P_{ir}
Responses from system level ($\mathbf{R}_s$)	$K_{sf}, K_{sr}, K_{tf}, K_{tr}, C_{\alpha f}, C_{\alpha r}$

The TC design problem at the vehicle level is stated as follows.

$$P_v\text{: Minimize} \left\|\omega_{sf} - \omega_{sf}^U\right\| + \left\|\omega_{sr} - \omega_{sr}^U\right\| + \left\|\omega_{tf} - \omega_{tf}^U\right\| + \left\|\omega_{tr} - \omega_{tr}^U\right\| + \left\|\omega_p - \omega_p^U\right\| + \left\|k_{us} - k_{us}^U\right\| + \varepsilon_{\mathbf{R}} + \varepsilon_{\mathbf{y}}$$

with respect to

$$(\omega_{sf}, \omega_{sr}, \omega_{tf}, \omega_{tr}, \omega_p, k_{us}, a, b)$$
$$(K_{sf}, K_{sr}, K_{tf}, K_{tr}, C_{\alpha f}, C_{\alpha r}, P_{if}, P_{ir})$$
$$(\varepsilon_{\mathbf{R}}, \varepsilon_{\mathbf{y}}) = (\varepsilon_{R1}, \varepsilon_{R2}, \varepsilon_{R3}, \varepsilon_{R4}, \varepsilon_{R5}, \varepsilon_{R6}, \varepsilon_{y1}, \varepsilon_{y2})$$

where

$$\omega_{sf} = \sqrt{\frac{K_{sf}}{M_{sf}}} \quad \omega_{sr} = \sqrt{\frac{K_{sr}}{M_{sr}}} \quad \omega_{tf} = \sqrt{\frac{K_{tf}}{M_{usf}}}$$

$$\omega_{tr} = \sqrt{\frac{K_{tr}}{M_{usr}}} \quad \omega_p = \sqrt{\frac{K_p}{J}} \quad k_{us} = \frac{Mb}{LC_{\alpha f}} - \frac{Ma}{LC_{\alpha f}} \tag{5}$$

$$K_p = \frac{2(a+b)}{\dfrac{(K_{sf}+K_{tf})}{aK_{sf}K_{tf}} + \dfrac{(K_{sr}+K_{tr})}{bK_{sr}K_{tr}}}$$

subject to

$$\left\|K_{sf} - K_{sf}^L\right\| \le \varepsilon_{R1}, \left\|K_{sr} - K_{sr}^L\right\| \le \varepsilon_{R2}, \left\|K_{tf} - K_{tf}^L\right\| \le \varepsilon_{R3}$$
$$\left\|K_{tr} - K_{tr}^L\right\| \le \varepsilon_{R4}, \left\|C_{\alpha f} - C_{\alpha f}^L\right\| \le \varepsilon_{R5}, \left\|C_{\alpha r} - C_{\alpha r}^L\right\| \le \varepsilon_{R6}$$
$$\frac{1}{2}\left(\left(P_{if} - P_{if}^L\Big|_{vert}\right)^2 + \left(P_{if} - P_{if}^L\Big|_{corn}\right)^2\right) \le \varepsilon_{y1}$$
$$\frac{1}{2}\left(\left(P_{ir} - P_{ir}^L\Big|_{vert}\right)^2 + \left(P_{ir} - P_{ir}^L\Big|_{corn}\right)^2\right) \le \varepsilon_{y2}$$
$$a^{min} \le a \le a^{max} \qquad b^{min} \le b \le b^{max}$$

Target Cascading at the System Level

At the system level the problem is stated as in Eq. (6):

$$\begin{aligned}
\mathbf{P}_s:\ & \text{Minimize} \quad \left\|\mathbf{R}_s - \mathbf{R}_s^U\right\| + \left\|\mathbf{y}_s - \mathbf{y}_s^U\right\| + \varepsilon_{\mathbf{R}} + \varepsilon_{\mathbf{y}} \\
& \text{with respect to} \quad \tilde{\mathbf{x}}_s, \mathbf{y}_s, \mathbf{y}_{ss}, \mathbf{R}_{ss}, \varepsilon_{\mathbf{R}}, \varepsilon_{\mathbf{y}} \\
& \text{where} \quad \mathbf{R}_s = r_s(\mathbf{R}_{ss}, \tilde{\mathbf{x}}_s, \mathbf{y}_s) \\
& \text{subject to} \\
& \quad \left\|\mathbf{R}_{ss} - \mathbf{R}_{ss}^L\right\| \le \varepsilon_{\mathbf{R}}, \quad \left\|\mathbf{y}_{ss} - \mathbf{y}_{ss}^L\right\| \le \varepsilon_{\mathbf{y}} \\
& \quad \mathbf{g}_s(\mathbf{R}_s, \tilde{\mathbf{x}}_s, \mathbf{y}_s) \le \mathbf{0}, \quad \mathbf{h}_s(\mathbf{R}_s, \tilde{\mathbf{x}}_s, \mathbf{y}_s) = \mathbf{0} \\
& \quad \tilde{\mathbf{x}}_s^{min} \le \tilde{\mathbf{x}}_s \le \tilde{\mathbf{x}}_s^{max}, \mathbf{y}_s^{min} \le \mathbf{y}_s \le \mathbf{y}_s^{max}
\end{aligned} \tag{6}$$

The objective function minimizes the discrepancy between current system level responses $\mathbf{R}_s$ and the targets set at the upper (vehicle) level $\mathbf{R}_s^U$, as well as between system linking variables $\mathbf{y}_s$ and the targets set at the vehicle level $\mathbf{y}_s^U$. Therefore, $\mathbf{R}_s^U$ and $\mathbf{y}_s^U$ are determined by solving Eq. (2). Target deviation tolerances are minimized to achieve consistent design with minimum discrepancies between the subsystem level responses $\mathbf{R}_{ss}$ and the target responses $\mathbf{R}_{ss}^L$ from the subsystem design problem, as well as between the subsystem level linking variables $\mathbf{y}_{ss}$ and the target values $\mathbf{y}_{ss}^L$ from the subsystem design problem. Since the system level is located in the middle of the overall hierarchy, this formulation is the most comprehensive, capturing all interactions, through linking variables, target responses from the lower level (superscript L), and target responses from the upper level (superscript U).

In the current study, there exist four design models at the system level: models for the front and rear suspensions, and tire models for vertical and cornering stiffness (Figure 2). The TC system-level design problem for the front suspension model is stated as follows.

$$\begin{aligned}
\mathrm{P}_{s1}:\ & \text{Minimize} \quad \left\|K_{sf} - K_{sf}^U\right\| + \varepsilon_{R1} + \varepsilon_{R2} + \varepsilon_{R3} \\
& \text{with respect to} \quad (zsmax_f, K_{Lf}, K_{Bf}, L_{0f}, \varepsilon_{\mathrm{R1}}, \varepsilon_{\mathrm{R2}}, \varepsilon_{\mathrm{R3}}) \\
& \text{where} \quad K_{sf} = \mathrm{AutoSim}(zsmax_f, K_{Lf}, K_{Bf}, L_{0f}) \\
& \text{subject to} \\
& \left\|K_{Lf} - K_{Lf}^L\right\| \le \varepsilon_{R1} \qquad K_{Lf}^{min} \le K_{Lf} \le K_{Lf}^{max} \\
& \left\|K_{Bf} - K_{Bf}^L\right\| \le \varepsilon_{R2} \qquad K_{Bf}^{min} \le K_{Bf} \le K_{Bf}^{max} \\
& \left\|L_{0f} - L_{0f}^L\right\| \le \varepsilon_{R3} \qquad L_{0f}^{min} \le L_{0f} \le L_{0f}^{min} \\
& \quad zsmax_f^{min} \le zsmax_f \le zsmax_f^{max}
\end{aligned} \tag{7}$$

Table 2: Summary of responses and variables at the system level

Design problem	P_{s1}	P_{s2}	P_{s3}	P_{s4}
Responses ($\mathbf{R}_s$)	K_{sf}	K_{sr}	K_{tf}, K_{tr}	$C_{\alpha f}, C_{\alpha r}$
Local variables ($\tilde{\mathbf{x}}_s$)	$zsmax_f$	$zsmax_r$	N/A	N/A
System-level linking variables ($\mathbf{y}_s$)	N/A	N/A	P_{if}, P_{ir}	P_{if}, P_{ir}
Responses from subsystem level ($\mathbf{R}_{ss}$)	K_{Lf}, K_{Bf}, L_{0f}	K_{Lr}, K_{Br}, L_{0r}	N/A	N/A

For a given target value for suspension stiffness from the vehicle TC problem in Eq. (5), the objective is to minimize the discrepancy between target and response. As there is no linking variable at the subsystem level, the term for minimizing the linking variable deviation is not included in the objective function. Besides the original variable bound constraints for suspension design, additional deviation constraints from the subsystem level are included in the constraint set. Deviations for subsystem level responses $K_{Lf}^L, K_{Bf}^L, L_{0f}^L$ are constrained within tolerance. The TC design problem for rear suspension model P_{sys2} is same as the one for the front except that it has different variable bounds.

The tire was represented as a single spring in the half-car model in the vehicle. At the system level, two different aspects of the same tire analysis model, vertical and cornering, are considered, and for each aspect a TC design problem is formulated.

The design models for the vertical and cornering tire stiffness are described in the following equations Eq. (8) and Eq. (9).

$$\begin{aligned}
\mathrm{P}_{s3}:\ & \text{Minimize} \quad \left\|K_{tf} - K_{tf}^U\right\| + \left\|K_{tr} - K_{tr}^U\right\| \\
& \text{with respect to} \quad (K_{tf}, K_{tr}, P_{if}, P_{ir}) \\
& \text{where} \quad K_{tf} = 0.9((0.1839P_{if} - 9.2605)F_m + 110119) \\
& \qquad\quad\ K_{tr} = 0.9((0.1839P_{ir} - 9.2605)F_m + 110119) \\
& \qquad\quad\ F_m = \frac{9.81Mb}{a+b} \\
& \text{subject to} \quad P_{if}^{min} \le P_{if} \le P_{if}^{max} \\
& \qquad\qquad\quad\ P_{ir}^{min} \le P_{ir} \le P_{ir}^{max}
\end{aligned} \tag{8}$$

$$P_{s4}\text{: Minimize } \left\|C_{\alpha f} - C_{\alpha f}^U\right\| + \left\|C_{\alpha r} - C_{\alpha r}^U\right\|$$

with respect to $(C_{\alpha f}, C_{\alpha r}, P_{if}, P_{ir})$

where

$$C_{\alpha f} = F_m\left(-2.668\times10^{-6}P_{if}^2 + 1.605\times10^{-3}P_{if} - 3.86\times10^{-2}\right)\frac{180}{\pi}$$

$$C_{\alpha r} = F_m\left(-2.668\times10^{-6}P_{ir}^2 + 1.605\times10^{-3}P_{ir} - 3.86\times10^{-2}\right)\frac{180}{\pi}$$

$$F_m = \frac{9.81Mb}{a+b} \tag{9}$$

subject to

$$P_{if}^{min} \le P_{if} \le P_{if}^{max}$$

$$P_{ir}^{min} \le P_{ir} \le P_{ir}^{max}$$

In the tire models, the objective function is to minimize deviations for the front and rear tire stiffnesses (vertical K_{tf}, K_{tr} or cornering $C_{\alpha f}, C_{\alpha r}$) subject to variable bound constraints for the tire inflation pressures for the front and rear P_{if}, P_{ir}. The stiffness of the tire in the vertical direction is a function of the inflation pressures and the datum vertical load on the tire F_m that is a function of the tire distances *a* and *b* and the mass of the vehicle M. The inflation pressures for the front and rear tires are linking variables that are coordinated at the vehicle level as in Eq. (5).

Target Cascading at the Subsystem Level

The subsystem level problem is stated in Eq. (10): minimize the deviations for subsystem responses and subsystem level linking variables subject to subsystem design constraints. Formally,

$$P_{ss}\text{: Minimize}_{\tilde{\mathbf{x}}_{ss}, \mathbf{y}_{ss}} \left\|\mathbf{R}_{ss} - \mathbf{R}_{ss}^U\right\| + \left\|\mathbf{y}_{ss} - \mathbf{y}_{ss}^U\right\|$$

$$\text{where } \mathbf{R}_{ss} = r_{ss}(\tilde{\mathbf{x}}_{ss}, \mathbf{y}_{ss}) \tag{10}$$

subject to

$$\boldsymbol{g}_{ss}(\mathbf{R}_{ss}, \tilde{\mathbf{x}}_{ss}, \mathbf{y}_{ss}) \le \mathbf{0}, \quad \boldsymbol{h}_{ss}(\mathbf{R}_{ss}, \tilde{\mathbf{x}}_{ss}, \mathbf{y}_{ss}) = \mathbf{0}$$

$$\mathbf{x}_{ss}^{min} \le \mathbf{x}_{ss} \le \mathbf{x}_{ss}^{max} \qquad \mathbf{y}_{ss}^{min} \le \mathbf{y}_{ss} \le \mathbf{y}_{ss}^{max}$$

At the bottom of the model hierarchy, subsystem design variables are input to the analysis models r_{ss} returning responses to the subsystem level as output. In Eq. (10), the objective is to minimize the deviations between the subsystem responses $\mathbf{R}_{ss}$ and the targets set at the system level $\mathbf{R}_{ss}^U$, as well as between the subsystem linking variables $\mathbf{y}_{ss}$ and the targets from the system level $\mathbf{y}_{ss}^U$. Target deviation tolerance constraints are not introduced in Eq. (10) because there are no lower level design models that need to be coordinated.

At the subsystem level below the suspension model, the front and rear coil spring design models minimize the difference between target coil spring stiffness and the response generated by the spring design analysis model. The coil spring design model attempts to minimize an objective function that is a weighted sum of the difference between target and actual linear spring stiffness, bending stiffness, and free length, while satisfying the following constraints [Shigley and Mischke 1989].

- maximum shear stress with safety factor must not be exceeded
- spring must not fail in fatigue
- coil diameter and wire diameter must fall within specified bounds
- wire diameter must be greater than the pitch
- wire diameter to coil diameter ratio must be reasonable
- spring must not be fully compressed at maximum suspension travel

The detailed equations for coil spring design including the above constraints are given in the following Eq. (11). Target values for linear spring stiffness K_L, bending stiffness K_B, and free length L_0 are cascaded down from the system level. Subsystem design variables are the wire diameter *d*, coil diameter *D*, and pitch *p*. Once optimal design is found, the updated target values are returned to the system level. Design model for the subsystem front coil spring design is given in the following Eq. (11):

$$P_{sub1}\text{: Minimize } \left\|K_{Lf} - K_{Lf}^U\right\| + \left\|K_{Bf} - K_{Bf}^U\right\| + \left\|L_{0f} - L_{0f}^U\right\|$$

with respect to (D, d, p)

$$\text{where } K_{Lf} = \frac{Gd^4}{8D^3\left(\frac{L_{0f} - 3d}{p}\right)} \quad K_{Bf} = \frac{EGd^4}{16D(2G+E)}$$

subject to

$$(F_a + F_m) \times \left(\frac{8D}{\pi d^3} + \frac{4}{\pi d^2}\right) - \frac{S_{su}}{n_s} \le 0 \tag{11}$$

$$n_f - \left(\frac{S_{su}S_{se}\pi d^3}{8D}\right) / \left(\left(\frac{4D}{d} + 2\right) / \left(\frac{4D}{d} - 3\right)F_a S_{su} + \left(\frac{2D}{d} + 1\right) / \left(\frac{2D}{d}\right)F_m S_{se}\right) \le 0$$

$$p - d \le 0$$

$$D^{min} \le D \le D^{max} \qquad d^{min} \le d \le d^{max}$$

where L_0 is spring free length, G is modulus of rigidity of spring material, n_s is the factor of safety in shear, S_{su} is the maximum allowable shear stress, S_{se} is fatigue endurance limit, and F_a, F_m are alternating and mean component of spring load.

This concludes our discussion of the formal statement of the problem. The next section illustrates the results of the process and explores the effects of changing target values, target weights, and design constraints.

5. DESIGN SCENARIO ANALYSIS

The computational process used to solve the TC problem in this study was as follows (Figure 2): First, the top level vehicle design problem was solved and system level targets were cascaded. Second, four system-level problems were solved independently based on the targets assigned from the top level. Third, subsystem-level problems for the front/rear coil spring design were solved. Based on the subsystem-level responses, system-level design problems for front/rear suspension design were solved again and all the system-level responses and linking variables from the four system design problems were fed back to the top level, completing one iteration. This process was not used as a formal coordination algorithm. Rather, iterations were terminated when the deviation terms became smaller than tolerance ε. Typically this was achieved within ten iterations.

In principle, the final results upon convergence of the target cascading algorithm depend on the relative weights assigned to the targets, on the target values themselves, and on the constraint bounds. In a multidisciplinary design exercise, decisions about the relative importance of each target are made *a priori* and may require adjustment depending on the degree and nature of their incompatibility. High level discussion subsequent to unsatisfactory target achievement may also result in constraint relaxation and thus a different design space. These issues are examined in light of the chassis design problem results.

Design Scenario A: Equally Weighted Ride and Handling Targets

The baseline study attempted to satisfy all design departments involved by assigning equal weight for each ride and handling target after scaling. Deviation quantities were scaled to the same order of magnitude to provide a meaningful comparison. Equal weights were used.

The target values and responses from the baseline study are given in Table 3. Figure 5 shows normalized comparisons of targets and responses, where “1” denotes an exact match, greater than 1 denotes exceeding the target, and less than 1 denotes not reaching the target. Exceeding the target does not necessarily mean better design, i.e., better than expected, because responses are normalized and the closer the response value is to 1, the better the target match. The optimal design from scenario A is given in Table 4. It is shown that TC yielded a consistent design such that for a given design quantity, such as front suspension stiffness, that was cascaded down from level *i* to level (*i*+1) as a target; the response from the analysis model at level (*i*+1) for that design quantity matched the target closely, within a tolerance. Similarly, linking variables converged to a single value within tolerance for each system they affected. If the tolerances were tightened, then the responses and linking variables would have matched more closely. Note that in Table 4 some of the variables were hitting lower or upper bounds, for example, the lower bound for the front suspension stiffness was active (boldface). This suggests that if the variable bounds were relaxed, then overall response would be changed for better achievement of targets. Different design scenarios are discussed in the following sections.

The final response values matched the targets closely, with the exception of the pitch natural frequency and the understeer gradient. These quantities are both functions of the distances *a* and *b*, which were hitting variable bounds. In the following subsections, two different design scenarios B and C will be presented. Design scenario B used different target values, and design scenario C used a modified design space. Indeed when the “design authority” encounters discrepancies in the achievement of certain design targets, there are two options to exercise: (1) changing targets, or (2) changing the design space. The following two sections explore these two options.

Table 3: Vehicle targets and vehicle responses

Target	Desired value	Scenario A	Scenario B	Scenario C
Front suspension first natural frequency ω_{sf} [Hz]	1.20	1.11	1.11	1.056
Rear suspension first natural frequency ω_{sr} [Hz]	1.44	1.55	1.54	1.51
Front suspension wheel hop frequency ω_{tf} [Hz]	12.00	11.55	11.54	11.12
Rear suspension wheel hop frequency ω_{tr} [Hz]	12.00	11.55	11.54	11.50
Pitch natural frequency ω_p [Hz]	0.50	0.87	0.87	0.81
Understeer gradient k_{us} [rad/m/s^2]	0.00719	0.00610	0.00590	0.00597

Table 4: Baseline design scenario A

Supersystem Design	Initial Values	Optimal Values	Lower Bounds	Upper Bounds
CG distance to front [m]	1.32	1.25	1.25	1.39
CG distance to rear [m]	2.38	2.39	2.31	2.45
Front suspension stiffness [N/mm]	40	40.83	13.13	56.25
Rear suspension stiffness [N/mm]	40	40	25.7	40
Front tire stiffness [N/mm]	20	30	12.31	30
Rear tire stiffness [N/mm]	20	30	11.95	30
Front cornering stiffness [N/rad/10e-4]	10	10.36	4.81	12.88
Rear cornering stiffness [N/rad/10e-4]	10	8.96	4.81	12.88
Front Suspension System Design				
Linear coil spring stiffness [N/mm]	159	140	**140**	160
Spring free length [mm]	393.6	412	350	420
Spring bending stiffness [N-mm/deg]	82500	85000	80000	85000
Overall suspension stiffness [N/mm]		41.18	18.7	56.25
Suspension travel [m]		0.0974	0.05	0.1
Vertical Tire System Design				
Front Tire Inflation Pressure [kPa]	100	125.49	83	330
Rear Tire Inflation Pressure [kPa]	100	192.85	83	330
Front Vertical Tire Stiffness [N/mm]		30		
Rear Vertical Tire Stiffness [N/mm]		29.88		
Cornering Tire System Design				
Front Tire Inflation Pressure [kPa]	100	124.16	83	330
Rear Tire Inflation Pressure [kPa]	100	193.33	83	330
Front Cornering Stiffness [N/mm]		11.07		
Rear Cornering Stiffness [N/mm]		8.35		
Front Coil Spring Subsystem Design				
Wire diameter [m]	0.02158	0.024	0.005	0.03
Coil diameter [m]	0.15068	0.19	0.05	0.2
Pitch	0.07814	0.1	0.05	0.1
Linear coil spring stiffness [N/mm]		140		
Spring bending stiffness [N-mm/deg]		84999.26		
Rear Suspension System Design				
Linear coil spring stiffness [N/mm]	159	140	140	160
Spring free length [mm]	393.6	412.15	350	420
Spring bending stiffness [N-mm/deg]	82500	84976	80000	85000
Overall suspension stiffness [N/mm]		41.18	10.1	40
Suspension travel [m]		0.098	0.05	0.1
Rear Coil Spring Subsystem Design				
Wire diameter [m]	0.02158	0.024	0.005	0.03
Coil diameter [m]	0.15068	0.1901	0.05	0.2
Pitch	0.07814	0.1	0.1	0.05
Linear coil spring stiffness [N/mm]		140		
Spring bending stiffness [N-mm/deg]		84997.52		

linking variables

Design Scenario B: Modification of Design Targets

Given the results of the baseline study in design scenario A, the design authority must assess the acceptability of the responses. If a certain response, for example pitch frequency, is deemed too high, the target cascading can be reapplied either with a different target value (i.e., different objective function) or a different design space. In design scenario B, the target value for pitch frequency was decreased to 0.3 Hz in an attempt to increase the deviation between the target and the response value, possibly causing the TC process to reduce the final pitch frequency. No changes were made to the feasible design space.

Responses after changing the target value are given in Table 3, and plotted in Figure 5. In Figure 5 the pitch frequency is compared to the same target value from the baseline study. In other words, ratio "1" for pitch frequency means a perfect match with the 0.5 Hz target value from design scenario A. Design scenario B, changing the target value alone, led to a negligible change from the baseline design scenario A.

The fact that front suspension stiffness affects pitch frequency, and the lower bound for the suspension stiffness was active, suggests that relaxing the feasible domain for the suspension stiffness would lead to better achievement of the target. This was investigated in design scenario C.

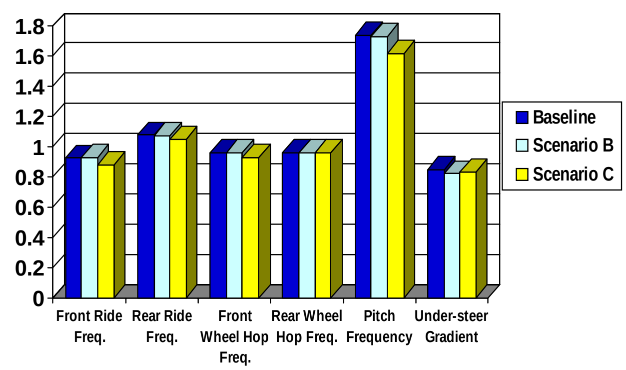

Figure 5. Comparison between design scenario A (baseline), B and C: "1" represents exact target match

Design Scenario C: Modification of Design Space

For design scenario C, target values were kept the same as in the baseline design scenario A. Instead, the variable bounds for coil spring stiffnesses in the front and rear suspensions were relaxed. A hypothetical design authority, upon receiving feedback from the baseline design, would realize that the targets assigned for each department were not achievable within the initial design space. Also, in the case of boundary optima, changing target values would not help to produce a better design in terms of achieving targets closely. The designers may then be allowed to change design specifications by changing the feasible space, material, or configuration.

The current case study changed the feasible space by relaxing variable bounds for certain design variables. As a result, the response (pitch frequency) that had the most significant discrepancy from the target value now had a response closer to the target value compared to design scenario B. The lower bound on front coil spring stiffness was still active despite being relaxed, raising the possibility that simultaneous attainment of all targets was not feasible for this SUV chassis design exercise within a reasonable design space.

Discussion

Both scenarios B and C led to a discrepancy for the fifth target, the pitch frequency, with slightly better results under scenario C with the modified design space. Changing targets (scenario B) did not have much effect on the response after the TC process because some of the constraints were active and some variables were bounded by variable bounds. Although the target was not closely matched, changing the design space (scenario C) had more effect on the response in terms of achieving the targets.

The design authority and teams can learn from these scenarios that when there is a discrepancy for certain targets, it is critical to have alternative design options rather than assigning new target values. Alternative design options include changing the design space, designing with different materials, or changing the design configuration.

The four system level design problems could be solved in a parallel fashion, but in this case they were solved sequentially, maintaining independent solution processes for each problem. Comparing computational efficiency for each scenario was not considered in the current study.

6. CONCLUSION

Analytical target cascading provides a rich framework for addressing large-scale, multi-disciplinary system design problems with a multilevel structure. Responses, linking variables, and local variables capture interactions between design problems and analysis models. From a design viewpoint, the main benefit of the proposed approach for target cascading is reduction in large-scale product design cycle time, avoidance of design iterations late in the development process, and increased likelihood that physical prototypes will be closer to production quality. The main difficulty is obtaining the appropriate analysis models. Convergence of the coordination strategy is an issue not addressed here but there is ample evidence that rigorous proof of convergence is less demanding than expected [Kim 2001].

7. ACKNOWLEDGMENTS

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