

## TARGET CASCADING: A DESIGN PROCESS FOR ACHIEVING VEHICLE RIDE AND HANDLING TARGETS

D. Geoff Rideout  
 Graduate Student  
 Automated Modeling Laboratory  
 University of Michigan  
 drideout@engin.umich.edu

Jeffrey L. Stein  
 Professor  
 Automated Modeling Laboratory  
 University of Michigan  
 stein@umich.edu

John B. Ferris  
 Senior Technical Specialist  
 ZF Industries, Inc.

### ABSTRACT

Vehicle dynamics are well understood by both academic researchers and automotive industries. And while modeling and simulation tools are still underutilized, they are becoming more frequently used in the vehicle design process. However, there is still lacking an overall design methodology that can link and integrate in a systematic fashion the design tasks of individual components or systems such that the vehicle performs as intended with a minimal number of design iterations. A process called Target Cascading, applied in the early stages of vehicle design, might serve as this systematic design methodology. In this paper, Target Cascading is evaluated for its ability to propagate top-level design specifications down to specifications for various subsystems and components in a vehicle design problem. More specifically, general ride and handling targets are set for a vehicle and these are cascaded down through the suspension, tire pressure and spring design levels by partitioning the original problem into a hierarchical set of subproblems. At a given level, an optimization problem is formulated to minimize deviations from the proposed targets and thus achieve intersystem compatibility. A coordination strategy links all subproblem decisions so that the overall supersystem performance targets are met. Results are presented that demonstrate Target Cascading's utility in unearthing tradeoffs and incompatibilities among initial targets early in the vehicle development cycle. Throughout the paper, the Target Cascading process is compared to traditional vehicle design strategies for achieving ride and handling targets. Target Cascading appears to be a promising systematic technique for the design of vehicles to meet ride and handling specifications.

### NOMENCLATURE

$C_{of}$  tire lateral cornering stiffnesses for front  
 $C_{or}$  tire lateral cornering stiffnesses for rear  
 $K_{sf}$  stiffness of front suspensions  
 $K_{sr}$  stiffness of rear suspensions  
 $K_{tf}$  stiffness of front tires  
 $K_{tr}$  stiffness of rear tires  
 $P_{if}$  front tire inflation pressure  
 $P_{ir}$  rear tire inflation pressure

$P_v$  vehicle level Target Cascading optimization problem  
 $P_s$  system level Target Cascading optimization problem  
 $P_{ss}$  subsystem level Target Cascading optimization problem  
 $R^L$  target values of  $R$  from a lower level  
 $R^U$  target values of  $R$  from an upper level  
 $R$  responses computed by analysis models  
 $T$  design targets  
 $a$  distance from vehicle center of mass to front axle  
 $b$  distance from vehicle center of mass to rear axle  
 $f$  objective for the design problem  
 $g$  inequality constraints for the design problem  
 $h$  equality constraints for the design problem  
 $k_{us}$  understeer gradient  
 $r$  response function  
 $u$  vehicle forward velocity  
 $x$  vector of all design variables  
 $\tilde{x}$  local design variables  
 $x^{\min}$  lower bound of  $x$   
 $x^{\max}$  upper bound of  $x$   
 $y$  linking design variables  
 $y^L$  target values of  $y$  from a lower level  
 $y^U$  target values of  $y$  from an upper level  
 $\epsilon_R$  target deviation tolerance for responses  
 $\epsilon_y$  target deviation tolerance for linking variables  
 $\omega_p$  pitch natural frequency  
 $\omega_{sf}$  first natural frequency of front suspension  
 $\omega_{sr}$  first natural frequency of rear suspension  
 $\omega_{tf}$  second natural frequency (wheel hop frequency) of front suspension  
 $\omega_{tr}$  second natural frequency (wheel hop frequency) of rear suspension  
 $z_{smax}$  suspension deflection at jounce bumper contact

### INTRODUCTION

Much of the motivation for this work comes from recent efforts in the automotive industry to formalize the product development process and to take better advantage of computer aided engineering (CAE)

tools. Balancing the many requirements for vehicles from many different engineering functions requires constant communication. Experts in various areas such as crashworthiness, mass analysis, ride and handling, as well as many other areas must continuously be aware of each other's designs (or proposed designs) to evaluate the design impact on their own area of responsibility. Designs are created concurrently, by independent teams of specialists with little interaction. However, work performed in isolation always requires re-evaluation by the rest of the organization, and decisions must be made quickly. Therefore, the quicker the communication of detailed information about a design proposal, the quicker will be the response from the rest of the organization on the side-effects.

An important part of creating a system design quickly is to identify the key links among design tasks, establish the appropriate criteria, identify constraints and limitations, and then let the individual functions determine the details. Subsequent interactions can take place when the specifications given to each task turn out to be difficult or impossible to meet, and another round of joint decisions is necessary.

The Target Cascading process [Kim, 2001] is a formal multilevel design optimization technique that has many features to allow large systems such as vehicles to be designed as described above. The important specifications for the entire system as well as for each system element (subsystems and components) are identified first, particularly those that will have influence on other parts of the system. Then values (targets) are assigned at the top level, usually based on input from marketing and program managers. These targets are then "propagated" to the rest of the system and appropriate values are assigned to each element of the system. Design tasks are then executed for each individual element, and interaction with the rest of the system is revisited only when a target cannot be met. One expects that the targets set are achievable by the particular system element within its design space, but consistency is another issue. If the value of a variable such as suspension stiffness, required to satisfy a ride and handling target, is not compatible with the suspension stiffness required to satisfy another target; then the design is not consistent. Target satisfaction within one level or group is usually well understood by the individual specialists, but consistency across the vehicle provides a greater challenge. This is one of the important functionalities provided by Target Cascading.

The design of vehicles, in this specific context to achieve ride and handling targets, is an example where a Target Cascading approach would be of particular value. The traditional process may be described in generic terms as follows. The design engineers receive some basic packaging constraints from the industrial designers, as well as functional objectives (ride and handling targets) from a development group. Concurrent design occurs at this stage as the industrial designers' constraints, based on their styling design effort, must be compatible with the development team's requirements for adequate space for suspension components and the like. Once fundamental design decisions are made (hard points, etc.) and test mules become available, the conversion of the ride and handling-related parameters into exact suspension spring and damper rates is done by the development group. If functional objectives cannot be met, the development group must work with the designers to achieve a solution or acceptable compromise. Downstream iterations are thus not necessarily disastrous, but once design has proceeded to test track

evaluation, it becomes very difficult to change specifications such as the location of the hard points.

In contrast, through Target Cascading, vehicle specifications can quickly and easily be used to synthesize system designs and component properties on multiple levels. Target Cascading allows the engineer to synthesize a new design and create new specifications very quickly in the event of changes in constraints or vehicle properties. Therefore, an engineer responsible for suspension design or ride & handling performance could immediately respond to proposed changes from other areas with a new optimized design proposal.

This purpose of this paper is to present a vehicle ride and handling design problem in the formal framework of the Target Cascading process. That is, in a vehicle context, high-level vehicle targets can be propagated downwards through the lower levels to generate additional specifications for suspension components, tires and suspension springs (these are the links with the other engineering departments and suppliers). The components, when assembled, must create a vehicle system for which the values of the ride quality and handling metrics deviate minimally from the original vehicle targets. The example was previously solved in [Kim et al., 2001] as the first demonstration of the formalism as a general design tool capable of coordinating and solving large-scale problems. In this paper Target Cascading is introduced as a relevant tool for the vehicle design community. The process, with the aid of the example, is shown to address the specific challenges presented by large-scale design artifacts in the automotive industry.

The next section presents a general description of the steps involved in the Target Cascading process. Section 3 develops the vehicle design example problem, describes the models, and qualitatively discusses the flow of target, response, and linking variable information. Section 4 gives the formal mathematical statement of the problem, and casts the vehicle design example in rigorous form. Sections 5 and 6 present and discuss results.

## TARGET CASCADING PROCESS

Target Cascading in vehicle design can be viewed as a four-step process: (i) specify overall vehicle mission targets, (ii) propagate vehicle targets to subsystem and component sub-targets, (iii) design vehicle systems, subsystems and components to achieve their respective sub-targets, and (iv) verify that the resulting design meets overall vehicle mission targets (see Kim et al. 2001). To set up this Target Cascading process the vehicle system must be partitioned into subproblems, the subproblems ordered into a hierarchy and linked by a coordination strategy and, finally, models of appropriate complexity for each subproblem established.

### System Partitioning

In general decomposition methodologies partitioning can be done in several ways, such as object, aspect (or discipline), and model-based [Wagner 1993]. Object and aspect partitioning are "natural" partitions and typically large companies employ both types of partitions simultaneously in a matrix organization. For example, an automotive manufacturer partitions its organization into powertrain, body, chassis, or electronics divisions (object), but has also dedicated groups for durability, packaging, dynamics, safety, or noise-vibration-harshness (aspect).

Model-based partitioning methods are formal mathematical procedures that divide large models of a system into smaller, more manageable models. After partitioning, design variables are categorized into linking variables, common to more than one subproblem, and local variables belonging only to one subproblem. In the Target Cascading formulation the easiest way is to start with an object partition and recognize that each design problem at a given level is likely to be multidisciplinary. The exact partitioning choice will also depend on the availability of models, so this task should be done carefully and should be considered as subject to revisions during process implementation.

### **Embodiment Design and Model Selection**

Having design models, and relatively simple analysis models available near the beginning of the process so that specific targets can be efficiently and easily evaluated is a key issue to the process. Evidence indicates that large scale models are often too time consuming to develop (or adapt to the new system), and too computationally intensive to be useful during the highly interactive design process. Because Target Cascading has features that address both the linking of information between “departments” as well as the use of design models to evaluate if targets are being met, many evaluations of the models are necessary. Thus, ideally, the models must be as simple as possible and computationally efficient. The models, without unnecessary detail, must be able to capture the salient characteristics of the system and system interactions, and must include the relevant design parameters. Such models have been called Proper Models [Wilson and Stein, 1994, Louca et al., 1997].

In many instances more than one model of the “same” system must be generated. That is, simple models used in the Target Cascading process at a given level are replaced by more detailed models for embodiment design at the level below. For example, a simple ride and handling target may be realized by changing the suspension stiffness in a half-car analysis model. The suspension stiffness then becomes a target for the design of the suspension system, one level down in the hierarchy. The suspension in the half-car model may be approximated by a translational spring and damper in parallel. One level below, the suspension is replaced by a more complex, kinematic representation with more design variables (e.g. link attachment points and coil spring stiffness) that are optimized to achieve the overall stiffness target from above. If current design targets cannot be realized in the more detailed models (i.e., the cascading problem is infeasible), the designer must either explore relaxing the local constraints or request adjustments at higher levels.

In the presentation of the Target Cascading formalism below, appropriate models are derived from the existing literature without a formal claim that they are the most appropriate (proper) models. Rather, the purpose of this work is to demonstrate the ability of Target Cascading to generate concurrent and efficient designs, and to assess the compatibility of high-level vehicle design targets. The models are appropriate to serve the illustrative example.

### **Model Hierarchy and Coordination Strategy**

After partitioning the original problem into subproblems in multiple levels, the linking variables between subproblems at the same level and the responses linking subproblems at different levels must be identified. A coordination strategy is required to ensure convergence of the solution generated by subproblem optimizations to the solution

of the original design problem. In a general hierarchical coordination strategy there is a master problem and one or more subproblems. The master problem is solved for the linking variables that are then input as parameters to the subproblems, and the subproblems are solved for local variables that are input as parameters to the master problem.

The Target Cascading hierarchy contains analysis models and design models. Each design model is an optimization problem that receives targets from higher levels. The design model optimizes design variables (dynamic system parameters) in the appropriate analysis model(s) such that the analysis model response deviates minimally from the targets. The term “response” throughout the paper refers to the output of an analysis, or simulation model. The design model then passes the optimal analysis model variable values down as targets to the level below.

The steps described above are not necessarily simple to execute, but they are systematic. Forcing design teams to create correct models and to negotiate selection of targets is essential for a successful process. The optimization formalism and attendant numerical solutions help put any further needed negotiations on a rational basis.

## **VEHICLE DESIGN EXAMPLE PROBLEM**

The design example is based on a production sport-utility vehicle (SUV). The targets and models were selected with an eye towards demonstrating the potential of Target Cascading as a design tool, and demonstrating how a vehicle design problem may be rigorously cast in the formulation. The models and targets, while a simplification of the vehicle design process as it occurs in practice, are not unrealistic and are appropriate for illustration. The primary goal in constructing this example is to achieve breadth and depth of the system hierarchy while maintaining manageable scope for solution on a desktop computer in a reasonable amount of time.

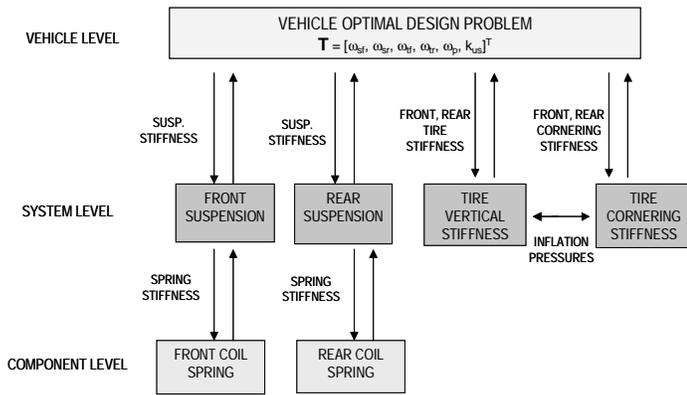
### **Vehicle Partitioning and Analysis Model selection**

For this example, the Target Cascading problem is partitioned as shown in Figure 1. The vehicle level, or “supersystem”, represents the whole car behavior from a ride and handling point of view. The next level down is termed the “system level”, representing separately the suspension and tires. Finally at the lowest level, the “component level”, the primitive parts such as springs are shaped, sized and evaluated. This may represent the involvement of a supplier. As will be shown below, the supersystem is decomposed into sprung mass, suspension, tires, and suspension springs (object decomposition). The vehicle-level model is also decomposed into ride and handling aspects, and the tire is partitioned into vertical and cornering stiffness aspects (aspect decomposition). The details including the analysis models, response and linking variables, and design optimization strategies will now be discussed for each level.

#### **Vehicle Level**

**Targets:** At the vehicle (supersystem) level the ride quality and handling targets are as follows:

- first natural frequency of front and rear suspension ( $\omega_{sf}$ ,  $\omega_{sr}$ )
- second natural frequency (wheel hop frequency) of front and rear suspension ( $\omega_{tf}$ ,  $\omega_{tr}$ )
- pitch natural frequency ( $\omega_p$ )
- understeer gradient ( $k_{us}$ )



**Figure 1 – Vehicle Design Problem Schematic**

Formulation of a simple all-encompassing ride quality target is difficult when one considers the historical difficulty in quantifying objective measures of ride quality and the sheer number of factors that contribute to the driver’s perception of ride [Ferris, 1999]. Use of the natural frequencies as targets is valid, given that for different classes of vehicles there exist frequency values that are necessary conditions for desirable ride characteristics.

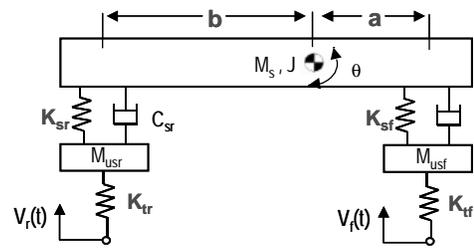
**Analysis Model:** The vehicle ride study is based on a half-car model. The target frequencies are functions of the sprung mass, unsprung masses, pitch moment of inertia, suspension and tire stiffnesses, and geometry. In this exercise the bounce and pitch modes are considered uncoupled and the reader is referred to Appendix A for the equations. A general half-car model is depicted in Figure 2 to illustrate the variables. The sprung and unsprung masses are assumed to be prescribed a priori, and thus are parameters as opposed to variables in design optimization terminology<sup>1</sup>. The major ride variables, i.e. the quantities that may be altered (within prescribed bounds) by the design engineer at the vehicle level, are the stiffnesses of the front and rear suspensions ( $K_{sf}$ ,  $K_{sr}$ ) and tires ( $K_{tf}$ ,  $K_{tr}$ ). The designer is also given a small amount of freedom to alter from a nominal value the distances  $a$  and  $b$  from the vehicle center of mass to the front and rear axles. Omission of the damping values of the shock absorbers will not appreciably affect the natural frequencies of the suspension, as the overall suspension damping ratio would in practice be on the order of 0.2 - 0.3. Clearly, however, for other ride targets, these design variables would be important.

The first natural frequencies of the suspensions are primarily affected by changing the front and rear suspension stiffnesses. To a lesser extent, modifying the distances  $a$  and  $b$  affects the natural frequencies by changing the portion of the sprung mass carried by the front and rear suspensions.

The wheel hop frequencies are functions mainly of the prescribed unsprung masses and the tire stiffnesses, which for a given tire vary as a function of inflation pressure ( $P_{if}$ ,  $P_{ir}$  front and rear, respectively) and normal vertical load.

<sup>1</sup> In the dynamics or system dynamics literature these would be called fixed parameters and parameters, respectively.

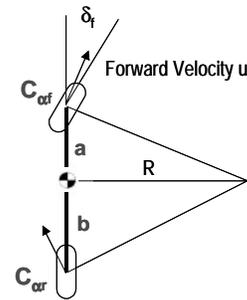
The pitch natural frequency is a function of the pitch moment of inertia (also prescribed), the distances  $a$  and  $b$ , and the suspension and (to a lesser extent) tire stiffnesses.



**Figure 2 – Half-Car Schematic**

The handling target, understeer gradient, is a measure of the direction and magnitude of the steering input (in addition to the Ackermann angle [Gillespie, 1992]) required for a vehicle to track a curve of constant radius  $R$  at a constant forward velocity  $u$ . Referring to the bicycle model of Figure 3, the understeer gradient is a function of  $a$ ,  $b$ , and the tire lateral cornering stiffnesses ( $C_{af}$ ,  $C_{ar}$  front and rear). For a given tire the cornering stiffness, like the vertical stiffness, varies as a function of inflation pressure and vertical load for small slip angles.

The half car and bicycle model equations were implemented in Matlab.



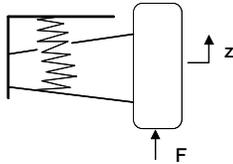
**Figure 3 – Bicycle Model**

At the vehicle level, the target estimates from the analysis model are termed vehicle-level responses. The system level target estimates, e.g. the value of  $K_{sf}$  estimated by the suspension analysis model and passed back to the vehicle level, are termed system-level responses to the vehicle level. The vector of variables at the vehicle level, for the purposes of the coordination algorithm, is partitioned into local design variables such as  $a$  and  $b$ , and response variables such as the stiffnesses. This nomenclature will be used extensively when the general mathematical problem statement is formally developed in the next section.

### System Level

**Targets:** The targets for the system level are suspension stiffnesses ( $K_{sf}$ ,  $K_{sr}$ ), tire vertical stiffnesses ( $K_{tf}$ ,  $K_{tr}$ ) and tire cornering stiffnesses ( $C_{af}$ ,  $C_{ar}$ ). These targets are determined in the course of solving the vehicle level problem to achieve the proper ride frequencies and understeer gradient.

**Analysis Models:** At the system level the suspensions and tires are modeled so that the suspension vertical stiffness, tire vertical stiffness, and tire cornering stiffness can be predicted. Analysis models of the front and rear suspensions are based on a three-dimensional multi-body model of an SUV short-long arm (SLA) suspension, constructed in AUTOSIM [Sayers, 1990, Hogland, 2001] and converted to an executable file. The configuration is shown in Figure 4. The suspension stiffness is calculated by incrementally applying a load to the spindle and measuring its vertical deflection. The load is incremented quasi-statically to render damping and inertial effects negligible. The design variables are the coil spring stiffness and free length. Adjusting these values alters the vertical suspension stiffness.



**Figure 3 – SLA Suspension**

Analysis models for the tires are taken from [Wong, 1997] and are presented in Appendix B. These models relate tire vertical and cornering stiffness to inflation pressure and normal load on the tire. These were coded in Matlab. The tire model contains two parts; one for vertical and one for the lateral characteristics of the tire. Because tire inflation pressure affects both of these characteristics, the local design algorithm must return a single value of tire pressure for the front tire, and a single value for the rear that gives the best possible values of vertical and cornering stiffness to minimize the deviation between prescribed and calculated vehicle targets. In target cascading terminology the tire inflation pressure represents a linking variable.

**Component Level:**

**Targets:** The targets for the component level are the translational and bending stiffnesses and the free lengths of the suspension coil springs. These targets are determined by the system level in its effort to meet the front and rear suspension overall stiffness targets.

**Analysis models:** At this level, the suspension coil springs are represented by a model taken from [Shigley, 1983]. The equations are given in Appendix C. The equations allow coil diameter, wire diameter, and coil pitch to be design variables from which are calculated vertical and bending stiffness of the coil spring. These equations were implemented in a Microsoft Excel spreadsheet.

**Model Hierarchy and Coordination Strategy**

Figure 1 gave a schematic of the example problem showing in general the flow of information between and across levels. At the vehicle level, the targets are prescribed. For a given vector of the design variables (a, b, and stiffness values), the half-car model and bicycle model equations generate a response vector. These are compared with the targets. The design variable values that generate an acceptable response are then passed to the system level as system level targets. For example, the optimization algorithm changes the front suspension stiffness  $K_{sf}$  in order to achieve the desired front suspension first natural frequency  $\omega_{sf}$ . Once an optimal value of  $K_{sf}$  is found, that value then becomes a target value for the suspension design problem, in which suspension variables (coil spring stiffness

and free length) are altered to achieve a suspension configuration with stiffness as close as possible to  $K_{sf}$ . The design variable values that give the optimal  $K_{sf}$  are then passed to the component level as targets, and spring geometry design variables are manipulated to produce a spring with suitable stiffness and free length.

Similarly, the optimal tire stiffnesses  $K_t$  and cornering stiffnesses  $C_{\alpha}$  calculated at the vehicle level become targets at the tire design portion of the system level.

Once the vehicle targets are cascaded down to the lowest level, the resulting design information must then be passed back up to the top level. In general at each level, it will not be possible to achieve the target values exactly, due to constraints on the local variables and the constraints on the inputs (design variables) that are passed up from lower levels. For example, when the optimization problem is solved at the suspension level to arrive at a coil spring stiffness and free length, constraints on these quantities may prohibit exact attainment of the target front suspension stiffness,  $K_{sf}$ . Or, upon passing the desired coil spring stiffness and length to the spring design model, packaging constraints and failure criteria at the component level may result in an optimal spring design with slightly different properties than desired. The deviation in spring stiffness results in a deviation in  $K_{sf}$ , which results in a deviation in first natural frequency of the front suspension.

The Target Cascading process in the present study was implemented in such a top-down, then bottom-up fashion. Starting from the vehicle level, targets were cascaded to the system level and then down to the subsystem level. Once the process reached the bottom level, responses were fed back to the system level and finally to the top vehicle level, completing one iteration loop. The four system level design problems could be solved in a parallel fashion, but in this case they were solved sequentially, maintaining independent solution processes for each problem. Iterations were executed until the deviations formally described in the next section fell within predetermined tolerances.

**TARGET CASCADING IN VEHICLE DESIGN: MATHEMATICAL FORMULATION**

In this section, the mathematical statement of the Target Cascading process is given for a general vehicle system partitioned into three subsystems or levels, here named the vehicle, system, and component levels.

**General Target Cascading Structure**

Figure 5 gives an example of the detailed description of the quantities and interactions at one particular level, in this case the system level. The two types of models in the Target Cascading process, *optimal design models*  $P$  and *analysis models*  $r$  are present. Targets for system responses and subsystem linking variables  $\mathbf{R}_{sys}^U$  and  $\mathbf{y}_{sys}^U$  are passed down from the vehicle level. After solving the system design problem, system responses and subsystem linking variables  $\mathbf{R}_{sys}^L$  and  $\mathbf{y}_{sys}^L$  are passed up to the vehicle level. Likewise, for subsystem 1,  $\mathbf{R}_{sub1}^U$  and  $\mathbf{y}_{sub1}^U$  are passed down as targets from the system-level design problem, whereas  $\mathbf{R}_{sub1}^L$  and  $\mathbf{y}_{sub1}^L$  are returned to the system level. Responses from subsystem 1,  $\mathbf{R}_{sub1}$ , local design variables  $\mathbf{x}_{sub1}$ , and linking variables  $\mathbf{y}_{sub1}$  are input to the analysis model  $r_{sub1}$  whereas responses  $\mathbf{R}_{sub1}$  are returned as output.

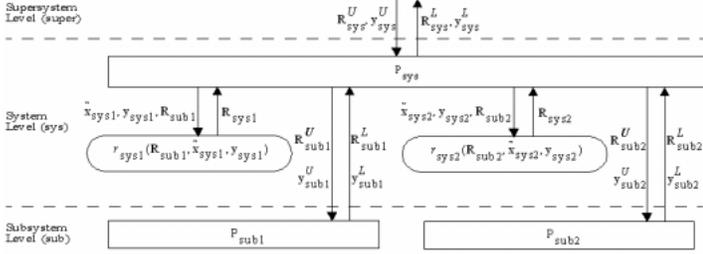


Figure 5 – System Level Details [Kim et al., 2001]

### Target Cascading at the Vehicle Level

At the top level of the vehicle hierarchy the problem is stated as follows: minimize the deviations between vehicle level responses and vehicle targets subject to vehicle design constraints and tolerance constraints that coordinate system responses and subsystem linking variables. Formally,

$$\begin{aligned}
 \mathbf{P}_v: \text{Minimize } & \|\mathbf{R}_v - \mathbf{T}_v\|, \mathbf{R}_v = \mathbf{r}_v(\mathbf{x}_v, \mathbf{y}_{sys}, \mathbf{R}_{sys}) \\
 & \mathbf{x}_v, \mathbf{y}_{sys}, \mathbf{R}_{sys}, \boldsymbol{\varepsilon}_R, \boldsymbol{\varepsilon}_y \\
 \text{subject to } & \|\mathbf{R}_{sys}^U - \mathbf{R}_{sys}^L\| \leq \boldsymbol{\varepsilon}_R, \quad \|\mathbf{y}_{sys}^U - \mathbf{y}_{sys}^L\| \leq \boldsymbol{\varepsilon}_y \\
 & \boldsymbol{\varepsilon}_R \geq 0, \boldsymbol{\varepsilon}_y \geq 0 \\
 & \mathbf{g}_v(\mathbf{x}_v, \mathbf{y}_{sys}, \mathbf{R}_{sys}) \leq 0 \\
 & \mathbf{h}_v(\mathbf{x}_v, \mathbf{y}_{sys}, \mathbf{R}_{sys}) = 0
 \end{aligned}$$

The first term of the objective function minimizes deviations between design targets  $\mathbf{T}$  and vehicle responses  $\mathbf{R}_v$ , where

$$\begin{aligned}
 \mathbf{T} &= [\omega_{sf}, \omega_{sf}, \omega_{sf}, \omega_{sf}, \omega_{sf}, k_{us}]^T \\
 \mathbf{x}_v &= [a, b]^T \\
 \mathbf{R}_{sys} &= [K_{sf}, K_{sf}, K_{tr}, K_{tr}, C_{of}, C_{of}]^T \\
 \mathbf{y}_{sys} &= [P_{if}, P_{tr}]^T
 \end{aligned}$$

Responses  $\mathbf{R}_{sys}$  are received from the system level. At the vehicle level  $\mathbf{R}_v$  is the analytical expression for the natural frequencies as expressed in Appendix A.

The objective function is augmented by adding deviation tolerances  $\boldsymbol{\varepsilon}_R$  to values of the responses from the system level, and  $\boldsymbol{\varepsilon}_y$  to values of the system linking variables. The norms of the deviations are constrained to be less than the tolerances. Constraints on values of  $a$  and  $b$  are enforced at this level. In this example a linking variable vector consisting of estimates of front and rear tire inflation pressure is passed to both the vertical tire spring and tire cornering stiffness models, and compared with the values returned from each system upon solution of the system-level optimization problem. At convergence, the deviation tolerance ideally becomes zero as the subsystem linking variables converge to the same values for the different subsystems. Finally,  $\mathbf{g}_v$  and  $\mathbf{h}_v$  are inequality and equality design constraints at the vehicle level.

In the example, the masses and inertia are fixed parameters with the following values:

- sprung mass 2282 kg
- unsprung mass (total) 228 kg
- pitch moment of inertia 6785 kg-m<sup>2</sup>.

Table 1 lists the lower and upper bounds for the local design variables and responses.

The lower bounds for the front and rear suspension stiffnesses are based on a maximum fully laden static deflection of 38 mm (1.5 in.) for the front and 89 mm (3.5 in.) for the rear suspensions. The vehicle level constraint set  $\mathbf{g}_v$  consists of the upper and lower bounds for  $a$  and  $b$ , and a constraint prohibiting the vehicle from undergoing a static pitch deflection of more than 2 degrees when fully laden. The fully laden condition is assumed to be a payload of 3 x 91 kg (200 lb.) 1 m aft of the center of gravity, and 227 kg (500 lb.) over the rear axle.

Table 1 - Variable Bounds - Vehicle Level

VARIABLE/RESPONSE	LOWER BOUND	UPPER BOUND
C.G. to front axle distance $a$ [m]	1.25	1.39
C.G. to rear axle distance $b$ [m]	2.31	2.45
Front suspension stiffness (one corner) $K_{sf}$ [N/m]	13100	56300
Rear suspension stiffness (one corner) $K_{sr}$ [N/m]	25700	60000
Front tire vertical stiffness [N/m]	120000	442000
Rear tire vertical stiffness [N/m]	120000	300000
Front tire cornering stiffness [N/rad]	67300	190000
Rear tire cornering stiffness [N/rad]	35000	100000

Lower and upper bounds on tire stiffness are calculated based on the lowest and highest inflation pressures in the experimental data and the normal loads on the front and rear tires at the vehicle trim condition.

### Target Cascading at the System Level

The four system design problems are formally stated as follows, in accordance with the symbols used in Figure 5. A complete treatment of the mathematical problem statement for all levels is found in [Kim et al., 2000 and 2001].

#### Front Suspension

$$\begin{aligned}
 \mathbf{P}_{Sys1}: \text{Minimize } & \|\mathbf{R}_{Sys1} - \mathbf{R}_{Sys1}^U\| + \boldsymbol{\varepsilon}_R \\
 \text{subject to } & \|\mathbf{R}_{Sub1} - \mathbf{R}_{Sub1}^L\| \leq \boldsymbol{\varepsilon}_R, \boldsymbol{\varepsilon}_R \geq 0
 \end{aligned}$$

where  $\mathbf{R}_{Sys1} = [K_{sf}]$ . The local design variable  $x_{Sys1}$ , front suspension travel  $z_{maxf}$  [m], is defined as the suspension deflection from the trim height to the onset of jounce bumper contact.  $\mathbf{R}_{Sub1}$ , the response vector from the spring design problem, consists of the linear coil spring stiffness  $K_{LINF}$  [N/mm], the bending stiffness of the coil spring

$K_{BENDf}$  [N-mm/rad], and the free length  $L_{Of}$  [mm]. Hogland [2000] has incorporated coil spring bending stiffness into the kinematic suspension model. Changing the free length of the spring changes the trim position of the nonlinear suspension, and thus affects both suspension stiffness and travel. The suspension travel is constrained to be between 0.19 m (7.5 in.) and 0.21 m (8.3 in.). The front and rear coil spring linear stiffnesses [N/m], bending stiffnesses [N-m/deg], and free lengths [mm] are constrained to fall within reasonable but somewhat arbitrary bounds, giving the following constraint set  $g_s$ .

- $0.190 \leq z_{smaxf} \leq 0.210$
- $120000 \leq K_{LInf} \leq 180000$
- $75000 \leq K_{BENDf} \leq 85000$
- $350 \leq L_{Of} \leq 420$ .

### Rear Suspension

$P_{Sys2}$ : similar to front suspension with the same suspension travel constraints. Quantities are denoted by the subscript  $r$  instead of  $f$ .

### Tire Vertical Stiffness

$P_{Sys3}$ : Minimize  $\|R_{Sys3} - R_{Sys3}^U\| + \|y_{Sys3} - y_{Sys3}^U\| + \epsilon_R + \epsilon_y$

where  $R_{Sys3} = [K_{Tr}, K_{Tr}]^T$ , the front and rear tire stiffnesses [N/m]; and  $y_{Sub3} = [P_{if}, P_{ir}]^T$ , the front and rear tire inflation pressures [kPa]. There is no constraint on responses from the subsystem level  $R_{Sub}$  because no subsystems exist directly below the tire models. The limits of the tire model data impose the following constraint set on the linking variables:

- $[83, 83]^T - [P_{if}, P_{ir}]^T \leq 0$
- $[P_{if}, P_{ir}]^T - [248, 248]^T \leq 0$

### Tire Cornering Stiffness

$P_{Sys4}$ : Minimize  $\|R_{Sys4} - R_{Sys4}^U\| + \|y_{Sys4} - y_{Sys4}^U\| + \epsilon_R + \epsilon_y$

subject to the same inflation pressure constraints, where  $R_{Sys4} = [C_{of}, C_{of}]^T$ , the front and rear cornering stiffnesses [N-m/rad]; and  $y_{Sub4} = [P_{if}, P_{ir}]^T$ , the inflation pressures. The values of inflation pressure returned to the vehicle level by the cornering stiffness design model will in general differ from the values generated by the vertical stiffness design model. Thus  $y_{Sub3}$  will not equal  $y_{Sub4}$ , but the discrepancy must be minimized to satisfy the convergence constraint at the vehicle level.

In the tire models the quantities  $a$  and  $b$  are passed down from the vehicle level as parameters. They are thus not treated as linking variables at the system level. The quantities are required to calculate normal force on the tires.

### Target Cascading at the Component Level

The component level problem is stated as follows:

$P_c$ : Minimize  $\|R_c - R_c^U\| + \|y_c - y_c^U\|$   
 $x_c, y_c$

where  $R_c = r_c(x_c, y_c)$

subject to  $g_c(x_c, y_c) \leq 0$   
 $h_c(x_c, y_c) = 0$

At the bottom of the model hierarchy, component design variables are input to the analysis models  $r_c$  returning responses to the subsystem level as output. Target deviation tolerance constraints are not introduced because there are no lower level design models that need to be coordinated.

The front and rear coil spring design models minimize the difference between the target vector  $R_c^U$  from the suspension level and the vector  $R_c$  generated by the spring design analysis model.

$P_{c1}$ : Minimize  $\|R_{c1} - R_{c1}^U\|$  (similar for component 2)

The coil spring analysis model attempts to minimize an objective function that is a weighted sum of the difference between target and actual linear and bending stiffnesses and free length, while satisfying the following constraints. Again, the reader is referred to Appendix C for details.

- maximum shear stress w/ safety factor must not be exceeded
- spring must not fail in fatigue
- coil diameter must be between 0.05 m and 0.2 m
- wire diameter must be between 0.005 m and 0.03 m
- wire diameter must be greater than pitch
- spring must have reasonable wire to coil diameter ratio
- spring must not be fully compressed at maximum suspension travel

The aforementioned constraints along with equality constraints that calculate stiffness values essentially combine the design and analysis models.

### Implementation

The Target Cascading problem was implemented in Matlab, and used Matlab's Sequential Quadratic Programming (SQP) optimization algorithm. The reader is referred to Papalambros and Wilde [1988] for details. Matlab's input/output capability allowed the optimizer to communicate with analysis models in the form of other Matlab m-files (vehicle and tire design), DOS executable files (suspension design), and Microsoft Excel spreadsheets (spring design). The Target Cascading process can thus occur across different application programs, across different computing platforms, and even over the Internet between departments or between automobile manufacturers and suppliers.

Specifics of the computational process used to solve the Target Cascading problem can be found in [Kim et al., 2001]. In principle, the final results upon convergence of the Target Cascading algorithm depend on the relative weights assigned to the targets, on the target values themselves, and on the constraint bounds. In a multidisciplinary design exercise, decisions about the relative importance of each target are made a priori and may require adjustment depending on the degree and nature of their incompatibility. High level discussion subsequent to unsatisfactory target achievement may also result in constraint relaxation and thus a different design space. These issues in the context of vehicle design are examined using the results in [Kim et al., 2001].

## RESULTS

### Case Study #1 - Equally Weighted Ride and Handling Targets

The targets were chosen based on accepted practice and the parameters of typical sport-utility vehicles. The first natural frequencies of the suspensions were chosen to provide attenuation of objectionable vertical vibration in the 4-8 Hz range. A rear natural frequency higher than the front aids in mitigating a vehicle's propensity to be set into pitching motion when the front wheels encounter a bump. The desired pitch natural frequency was set well below the 1-2 Hz fore-aft vibration frequency range that is correlated with occupant discomfort.

The first optimization run attempted to satisfy all the ride and handling objectives by equally weighting each target. Quantities were scaled to the same order of magnitude so that deviations in, for example, tire stiffness (on the order of 100000 N/m) and suspension stiffness (on the order of 10000 N/m) would contribute equally to the norm of  $T - R_v$ .

The target and response values from the first case study are given below in Table 2. Figure 6 shows normalized comparisons of targets and responses, where a value of 1 denotes an exact target match, greater than 1 denotes a higher than desired response magnitude (overshot target), and less than 1 denotes a less than desired response magnitude (undershot target).

The final response values match the targets very closely with the exception of the understeer gradient and the pitch natural frequency. These quantities are both dependent on distances a and b, which are 1.25 m (lower bound) and 2.45 m (upper bound) respectively upon termination of the algorithm. While the interrelations between variables become quite complex for even this simple system, one can immediately observe that maximizing the distance b relative to a would have maximized the understeer gradient for a given pair of tire cornering stiffnesses. Yet, the understeer gradient was less than desired. Minimizing both a and b in the absence of other factors minimizes the pitch natural frequency. There thus exists the potential for incompatibility between the two targets.

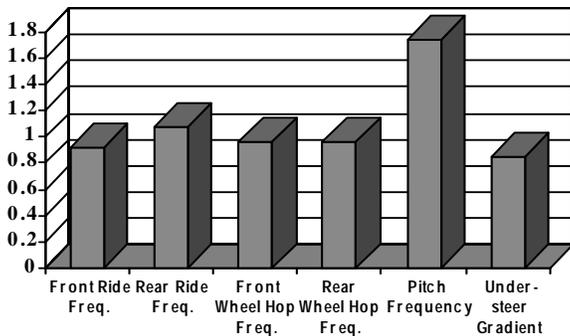


Figure 6 - Normalized Targets - Case Study #1

Table 2 - Vehicle Targets

TARGET DESCRIPTIONS	TARGET VALUE	RESPONSE VALUE
Front susp. first natural frequency $\omega_{sf}$ [Hz]	1.20	1.11
Rear susp. first natural frequency $\omega_{sr}$ [Hz]	1.44	1.55
Front susp. wheel hop frequency $\omega_{fr}$ [Hz]	12.00	11.55
Rear susp. wheel hop frequency $\omega_{rr}$ [Hz]	12.00	11.55
Pitch natural frequency $\omega_p$ [Hz]	0.50	0.87
Understeer gradient $k_{us}$ [rad/m/s <sup>2</sup> ]	0.00719	0.00610

### Target Cascading Design Consistency

Table 3 shows how the Target Cascading algorithm yields a consistent design such that for a given quantity, such as front suspension stiffness, that is passed down from the  $i^{\text{th}}$  level to the  $(i+1)^{\text{th}}$  level as a target, the response from the analysis model at the  $(i+1)^{\text{th}}$  level for that quantity closely matches the target. The reader can verify the similarity between the pairs of variables numbered 1 to 5. Linking variables (5) converged approximately to a single value for each system they affect. For example, the front tire inflation pressure for the tire vertical stiffness model is 124.8 kPa, (18.1 psi) and is 123.35 kPa (17.9 psi) for the cornering stiffness model. In the interests of brevity, Table 3 demonstrates the results of the design for the front of the vehicle; however, the design of the rear suspension, tires, and spring exhibited similar consistency. Table 3 also shows the upper and lower bounds for each quantity, with active bound quantities underlined. For example, the active lower bound of 120 N/mm for the front spring stiffness indicates that the optimization algorithm lowers the spring stiffness until prohibited from further doing so by the bound, and presumably would continue to lower the value if the bound is relaxed.

Table 3 – Case Study #1 Design Consistency

Variables	Optimal	Lower Bounds	Upper Bounds
<b>Vehicle</b>			
Front susp. stiffness [N/m]	36930	13130	56250
Front tire stiffness [N/m]	300000	123100	<u>300000</u>
Front cornering stiffness [N/rad]	105500	67300	190000
<b>Susp.</b>			
$K_{SPRING}$ [N/mm]	120	<u>120</u>	180
Front susp. stiffness [N/m]	37090	18700	56250
<b>Tire (Vert.)</b>			
Front Tire Pressure [kPa]	124.8	83	330
Front Tire Vert. Stiffness [N/m]	300000		
<b>Tire (Lat.)</b>			
Front Tire Pressure [kPa]	123.35	83	330
Front Cornering Stiffness [N/rad/m <sup>2</sup> ]	111000		
<b>Spring</b>			
Wire diameter [mm]	24.3	5	30
Coil Diameter [mm]	200	50	200
Pitch [mm]	97.2	50	100
$K_{SPRING}$ [N/mm]	120.4		

## Case Study #2 (Option A)

Having verified that the algorithm is well-behaved, the design team must assess the acceptability of the responses. If, for example, the pitch natural frequency response is deemed too high, but the understeer gradient is still within acceptable limits, the Target Cascading process can be reapplied with a different objective function or different design space. For the second case study, the target value of  $\omega_p$  is decreased to 0.3 Hz in an attempt to increase the deviation between the target and response value, possibly causing the algorithm to reduce the final response value. No changes were made to the design space.

Changing the target value alone had negligible effect on the responses and the final vehicle variable values. Bearing in mind the active lower bound on front suspension spring stiffness, and recognizing that as a rule of thumb lowering the spring (and thus front suspension) stiffness in relation to the rear reduces pitch natural frequency; an expansion of the design space by relaxing that bound is a potential solution. It should be remembered that it is assumed that the wheelbase may not be shortened to a value below the sum of the current lower bounds of a and b.

## Case Study #3 (Option B)

The Target Cascading process was repeated with an expanded design space in which the front coil spring stiffness lower bound was reduced from 120 to 100 N/mm. Further, the weight of the pitch natural frequency target was increased tenfold in the objective function. Results for case studies #2 and #3 are shown in Figure 7.

In case study #3 the pitch frequency was reduced from 0.87 to 0.79 Hz, with a corresponding drop in understeer gradient to 0.00677. The distances a and b were both minimized, and the front spring stiffness was reduced to the new lower bound of 100 N/mm. At this point if the design is still not acceptable, further investigation and discussion are required. A constraint restricting laden static pitch deflection to 2 degrees was also not active, suggesting that the constraint space can tolerate further lowering of the front suspension stiffness.

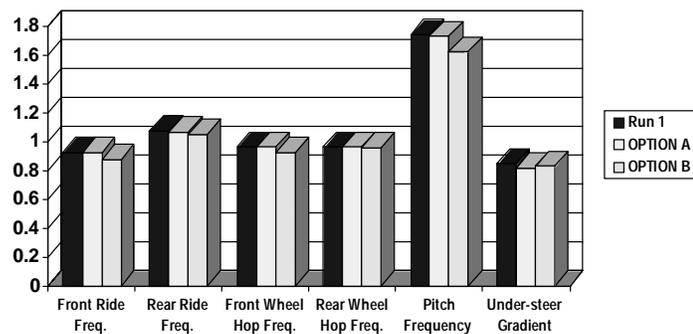


Figure 7 – Case Studies #1-#3

## DISCUSSION

The goals of this research were twofold. The first goal was to demonstrate that the Target Cascading formalism could be applied to the design of a vehicle to meet ride and handling specifications. Second, Target Cascading was to show how vehicle level specifications can be cascaded down to system and component levels

and that subsequent changes that occur at these levels determine the extent to which specific system level specifications are consistent with other design team constraints.

From a design viewpoint, the main potential benefits of the proposed approach for Target Cascading are reduction in vehicle design cycle time, avoidance of design iterations late in the development process, and increased likelihood that physical prototypes will be closer to production quality. Design iterations are reduced by integrating the target propagation and target matching processes into a single procedure. Using a partitioning comprised of systems, subsystems, and components reduces the complexity of the overall design problem and allows more systematic concurrent design of the system's elements.

The multiple application programs used in the simple vehicle example illustrate the potential for Target Cascading to occur across different computing platforms or across different departments or companies via the Internet. It is not necessary for everyone to represent their products in the same way using the same software, rather only that certain interface conventions be observed. This should reduce the resistance to this approach that might otherwise occur when design teams are asked to replace their existing representations (models) and software.

While previously developed models can be utilized, the potential efficiency, consistency and robustness of the Target Cascading process is dependent on the use of appropriate models. This means that multiple representations of the same objects will be needed. In this example, two models of the suspension system were used. A challenge to the use of Target Cascading is that obtaining models of systems or components is a costly and lengthy process and often done without regard for or knowledge of what is actually required in the model to satisfy the current design needs. Clearly better modeling tools are necessary so that rapid creation of "proper" or appropriate models of appropriate complexity, known validity and physically meaningful parameters and variables can occur. Research is ongoing on this topic. See for example Louca et al. (2001).

Compared with the traditional vehicle design process, Target Cascading appears to be able to elevate the degree of concurrency with which different systems are designed and integrated. The algorithm can at any time receive target changes, compute the consequences to all departments by updating targets at each level of the hierarchy, and communicate those new targets. This is not to say that the existing process would be or can be replaced. Ride and handling metrics, for example, are not easily quantified - the test track phase and tuning of test mules remains essential to the achievement of a qualitatively satisfactory vehicle. The input of designers at this stage also ensures that the product is imbued with the visceral qualities desired by the target market.

Target Cascading appears to be able to expedite the process of setting new targets when old targets are unrealizable. Traditionally, realization that, e.g., a mass target could not be met, required that an updated mass target be generated and then communicated to all parties affected, such as ride and handling teams. The parties would then perform new analyses independently and report back on their degree of success in creating new acceptable designs. With Target Cascading, all analysis models are under a single umbrella and communicate with each other. The rest of the organization can immediately begin

analyses on their functions. The individual design tasks can be run concurrently with the new target mass, and the feasibility of generating a new consistent vehicle design can be evaluated.

The Target Cascading results must be interpreted in light of parameter uncertainty. For example, the optimal front tire pressure in the case study was 124 kPa (18 psi). One must assume that the pressure will vary during normal vehicle usage due to such factors as ambient temperature change. The effect on the ride and handling targets of such a variation from the optimal parameter value is an important consideration. At present Target Cascading has not been combined with formal "design for uncertainty" methods; however, the simulation environment allows the designer to quickly assess the consequences of the parameter drifting to the nominal optimal value plus or minus some reasonable tolerance. If the anticipated real tire pressure range was 14-22 psi, then the Target Cascading exercise could be repeated, but with the inflation pressure fixed at the upper or lower bound as opposed to being a linking variable. The resulting variation in the overall design targets can then be evaluated.

In the current example, the tire models are somewhat trivial and suggest that an existing tire type was set with only the inflation pressure provided as a design parameter by which to give the best compromise between vertical and cornering stiffness. The authors realize that this is an oversimplification but included the tire model in this way to demonstrate the algorithm's ability to handle linking variables. The process, in practice, could generate desired tire parameters such as vertical and cornering stiffness, and thus provide unambiguous targets for suppliers. The coil spring targets also represent the sort of information that could be communicated to the supply chain through a two-way communication path. Throughout the vehicle design process, decisions can be made concurrently with suppliers, taking into consideration their existing inventory or possible custom-made components.

Future work is needed to determine systematically the required model complexity for each level of a Target Cascading design problem. If an inventory of models of varying complexity is assembled at each level, and a means of selecting the appropriate models is devised, then a Target Cascading problem can be solved accurately with minimal computational effort. Target Cascading subproblems could be nested within elements of a larger problem. Further research is also required to articulate quantifiable targets for areas such as NVH, durability, and manufacturability.

## SUMMARY AND CONCLUSIONS

Target Cascading as a generic design framework for solving large-scale, multi-disciplinary system design problems with a multilevel structure was applied to a vehicle design. In particular ride and handling targets based on accepted metrics were determined, and propagated down through system and component levels by a successful Target Cascading process. Vehicle ride and wheel hop frequencies were attainable targets within the design space bounded by constraints such as wheelbase and laden static deflection. The desired pitch natural frequency was not compatible with the other targets and the constraints. The Target Cascading solution pointed out design space changes that, subject to the approval of groups involved, would aid target attainment.

The successfully solved example gave a consistent design, unearthed design target incompatibilities, allowed quick study of the

effect of changing target priorities, and assessed the impact of changes in the design space. While the vehicle system included only passive elements, there is no reason to believe that active elements and design of their control schemes is incompatible with the Target Cascading process as long as models exist from which responses can be generated as a function of design variables. Study of partitioning issues unique to active systems remains as future work.

The use of many models in different software environments indicates the potential for Target Cascading to be performed over the Internet. This would clearly facilitate the linking of departments within companies and increase the concurrency of design, as well as involve the suppliers in addition to the automotive companies.

In conclusion, it appears that Target Cascading represents a viable tool for vehicle design, in particular, for vehicle design that includes dynamic systems. The advantages that would accrue to the company utilizing a Target Cascading approach to solve large-scale design problems will increase as the state of the art advances in vehicle system simulation, and as targets for subjective areas such as NVH and ride quality become more accurately quantified. Future work strives to increase the efficiency of the algorithm by systematically generating the model of appropriate complexity for each level of the hierarchy.

## ACKNOWLEDGMENTS

The authors would like to acknowledge the generous sponsorship of the U.S. Army Automotive Research Center. The work of Hyung Min Kim of The University of Michigan in development and computer implementation of the algorithm, and the industry perspective provided by Derek Hogland of General Motors Corporation, were invaluable and greatly appreciated.

## REFERENCES

- Ferris, J., "Factors Affecting Perceptions of Ride Quality in Automobiles," 1999, ASME International Conference and Exposition.
- Gillespie, T.D., 1992. *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers, Warrendale, PA.
- Hogland, D., 2001. *A Parametric Model to Generate Subsystem Constitutive Laws for a Vehicle Ride Model*, SAE Paper 01PC-203.
- Kim, H. M., *Target Cascading in Optimal System Design*, 2001, Doctoral Dissertation, University of Michigan.
- Kim, H. M., Michelena, N. F., Papalambros, P. Y. and Jiang, T., "Target Cascading in Optimal System Design," 2000, *Proceedings of ASME DETC 2000*, DETC2000/DAC-14265.
- Kim, H. M., Rideout, D.G., Papalambros, P.Y., Stein, J.L., "Analytical Target Cascading in Automotive Vehicle Design," 2001, Submitted to ASME DETC 2001, 27<sup>th</sup> Design Automation Conference.
- Louca, L.S., Stein, J.L., Hulbert, G.M., and Sprague, J., "Proper Model Generation: An Energy-Based Methodology," 1997, *Proc. ICBGM '97*, Phoenix, AZ.
- Louca, L.S., Stein, J.L., and Rideout, D.G., 2001. "Generating Proper Integrated Dynamic Models for Vehicle Mobility Using a Bond Graph Formulation." *Proc. ICBGM '01*, Phoenix, AZ.
- Papalambros, P. Y. and Wilde, D., 2000. *Principles of Optimal Design: Modeling and Computation* (2nd Ed.), Cambridge University Press, New York.
- Shigley, J. E. and Mischke, C. R., *Mechanical Engineering Design* (5th Ed.), 1989, McGraw-Hill Book Co.

Wagner, T.C., *A General Decomposition Methodology for Optimal System Design*, 1993, Doctoral Dissertation, University of Michigan.

Wilson, B.H. and Stein, J.L., *An Algorithm for Obtaining Proper Models of Distributed and Discrete Systems*, 1994, *J. Dynamic Systems Measurement and Control*, Vol. 117, No. 4, pp. 534-540.

Wong, J. Y., *Theory of Ground Vehicles* (2nd Ed.), 1993, John Wiley & Sons Inc.

## APPENDIX A – VEHICLE-LEVEL TARGET EQUATIONS

Referring to Figure 2, the bounce natural frequencies are calculated considering each end of the vehicle individually.

$$\omega_{sf} = \sqrt{\frac{K_{sf}}{M_{sf}}} \quad [\text{rad/s}] \quad \omega_{sr} = \sqrt{\frac{K_{sr}}{M_{sr}}} \quad [\text{rad/s}]$$

where  $M_{sf}$  and  $M_{sr}$  are the portions of the mass sprung by the front and rear suspensions.

Wheel hop frequencies:

$$\omega_{uf} = \sqrt{\frac{K_{uf}}{M_{usf}}} \quad [\text{rad/s}] \quad \omega_{ur} = \sqrt{\frac{K_{ur}}{M_{usr}}} \quad [\text{rad/s}]$$

where  $M_{usf}$  and  $M_{usr}$  are the front and rear unsprung masses.

Pitch natural frequency and stiffness:

$$\omega_p = \sqrt{\frac{K_p}{J}} \quad [\text{rad/s}]$$

$$K_p = \frac{2(a+b)}{\frac{(K_{sf} + K_{tf})}{a(K_{sf} K_{tf})} + \frac{(K_{sr} + K_{tr})}{b(K_{sr} K_{tr})}} \quad [\text{N-m/rad}]$$

Frequencies are calculated based on decoupled bounce and pitch modes.

Understeer gradient:

$$k_{US} = \frac{mb}{(a+b)C_{of}} - \frac{ma}{(a+b)C_{or}} \quad [\text{rad/m/s}^2]$$

## APPENDIX B – TIRE MODEL EQUATIONS

The stiffness of the tire in the vertical direction is a function of the inflation pressure  $P_i$  [kPa] and (for simplicity) the datum vertical load on the tire  $F_m$  [N]. The load due to vehicle mass is distributed between the front and rear tires as a function of the distances  $a$  and  $b$ . The coefficient 0.9 is an approximate adjustment to convert static stiffness to rolling stiffness. The remaining coefficients are from a curve fit to sample data from Wong [1993].

$$K_t = 0.9 * ((0.1839 * P_i - 9.2605) * F_m + 110119) \quad [\text{N/m}]$$

where

$$F_m = M * b * 9.81 / (a + b).$$

Similarly, the cornering stiffnesses depend on pressure and normal load:

$$C_\alpha = F_m * (-2.668 \times 10^{-6} * P_i^2 + 1.605 \times 10^{-3} * P_i - 3.86 \times 10^{-2}) * 180 / \pi \quad [\text{N/rad}].$$

## APPENDIX C – SPRING DESIGN EQUATIONS

The spring design module calculated linear stiffness and bending stiffness as a function of coil diameter  $D$ , wire diameter  $d$ , and pitch  $p$ . At the optimal design point, the difference between the calculated values and the optimal spring parameters from the suspension design problem was minimized.

From Shigley (1989), linear stiffness  $K_{lin}$ :

$$K_{lin} = \frac{Gd^4}{8D^3 \left( \frac{L_o - 3d}{p} \right)} \quad [\text{N/mm}]$$

where  $L_o$  is spring free length in mm,  $G$  is modulus of rigidity of spring material.

From Hogland (2000), spring stiffness in bending  $K_{bend}$ :

$$K_{bend} = \frac{EGd^4}{16D(2G + E)} \quad [\text{N-mm/rad}].$$

The following constraints incorporate the formulae for shear stress and factor of safety for fatigue failure. The first constraint requires that the maximum shear stress be less than the maximum allowable shear stress divided by the factor of safety in shear  $n_s$ . The second constraint ensures that the actual fatigue failure factor of safety is greater than the minimum allowable factor  $n_{fat}$ .

$$(F_a + F_m) * (8D/\pi d^3 + 4/\pi^2) - S_{su}/n_s \leq 0$$

$$n_{fat} - (S_{su} S_{se} \pi d^3 / 8D) / ((4D/d + 2)/(4D/d - 3) F_a S_{su} + (2D/d + 1)/(2D/d) F_m S_{se}) \leq 0$$

where  $S_{su}$  = maximum allowable shear stress  
 $S_{se}$  = fatigue endurance limit  
 $F_a$   $F_m$  = alternating and mean components of spring load.